# Supergeometry, supersymmetry and quantization

December 16-19, 2019, University of Luxembourg

## Talks

- **Albert Schwarz**
  Title: *Geometric approach to quantum theory*
  Abstract: In the geometric approach the starting point is the convex set of states. The rules for the calculation of probabilities can be derived from the first principles; they come from small adiabatic interaction with the environment. We discuss symmetries and supersymmetries in the geometric approach and in the language of Jordan algebras closely related to this approach.
  One can define the notions of particle and quasiparticle. The collisions of (quasi) particles in the are described by inclusive scattering matrix in the geometric approach. (The usual notion of scattering matrix cannot be applied to quasiparticles).

- **Hovhannes Khudaverdian**
  Title: *Thick morphisms and action in classical mechanics*
  Abstract: For an arbitrary morphism of (super)manifolds, the pull-back is a linear map of the spaces of functions. In 2014, Th.Voronov have introduced the notion of a "thick morphism" of (super)manifolds, which define a pull-back of functions that is generally non-linear. This construction was introduced as an adequate tool to describe $L_\infty$ morphisms of algebras of functions when they are provided with a structure of a homotopy Poisson algebra. It turns out that if you go down from 'heavens to earth' and consider usual (not super!) manifolds, we obtain constructions which have natural interpretation in classical and quantum mechanics. In particular, in this case the geometric object which defines a "thick diffeomorphism" becomes action of classical mechanics.
  I will define a thick morphism and will tell shortly about their application in homotopy Poisson algebras. Then I will discuss the relation of thick morphisms with action.
  The talk is based on the work:
  "Thick morphisms of supermanifolds, quantum mechanics and spinor representation", J. Geom. and Phys., 2019, article Number: 103540, DOI:10.1016/j.geomphys.2019.103540, arXiv:1909.00290

- **Janusz Grabowski**
  Title: *Contact Courant algebroids*
  Abstract: Departing from a specific definition of super-contact structures, we develop the concept of a contact Courant algebroid. Examples will be given.

- **Glenn Barnich**
  Title: *Remarks on BV-BFV quantization and soft degrees of freedom in the presence of boundaries*
  Abstract: After a quick review of Lagrangian BV and Hamiltonian BFV quantization and their relation, we analyze in the simplest case of the quantized electromagnetic field confined between two perfectly conducting plates, the set-up of the Casimir effect, the additional topological degrees of freedom that give rise to contributions to the partition function that scale with the area of the plates.

- **Urs Schreiber**
  Title: *Super-exceptional embedding construction of the M5-brane*
  The Green-Schwarz-type action functional for the supermembrane in 11d, i.e. for the fundamental M2-brane sigma model, turns out to elegantly arise as the supersymmetric trivialization of the C-field superspace 4-cocycle restricted from 11d super-space to the M2-brane's super-worldvolume. This is known as the "super-embedding construction". But an analogous construction of the M5-brane action had remained an open problem.
  In this talk I present a recent result (arxiv.org/abs/1908.00042) which shows that the M5-brane action, too, does arise from a super-embedding construction – but after first passing from plain 11d super-space to super-exceptional super-space. This is joint work with H. Sati and D. Fiorenza. Slides will be available at: ncatlab.org/schreiber/show/Super-exceptional+embedding+construction+of+the+M5-brane

- **Luca Vitagliano**
  Title: *The deformation cohomology of a VB-algebroid*
  Abstract: VB-algebroids are vector bundles in the category of Lie algebroids, and appear naturally in Poisson and related geometries. They can be also seen as vector bundles in the category of (degree 1) NQ-manifolds. In this talk, we extend the deformation theory of Lie algebroids by Crainic and Moerdijk to VB-algebroids. Namely, we describe a DG Lie algebra controlling deformations of a VB-algebroid in an appropriate sense and discuss its cohomology. The theory is illustrated via some examples. This is joint work with Pier Paolo La Pastina.

- **Pietro Grassi**
  Title: *Supermanifolds, Superstring Field Theory and A-Infinity-Algebra.*
  Abstract: Inspired by the analogy between different types of differential forms on supermanifolds and string fields in superstring theory, we construct new multilinear non-associative products of forms which yield, for a single fermionic dimension, an A-Infinity algebra as in string field theory. For multiple fermionic directions, we give the rules for constructing non-associative products, which are the basis for a full A-Infinity algebra structure.

- **Francesco Toppan**
  Title: *Nonassociativity and Physics*
  Abstract: In this talk I discuss how nonassociativity can be encoded in physical theories and which is the operational set-up needed for its detection. In mathematics, the nonassociative division algebra of the octonions is at the core of several exceptional mathematical structures. In physics, besides the single case of the exceptional Jordan algebra (within the Jordan's formulation of quantum mechanics) nonassociativity remains elusive. Speculations about an exceptional Theory of Everything (like the one offered by the octonionic M-algebra) require going beyond the Jordan's algebra framework. I focus on new results obtained via the "quasi-nonassociative" method of octonionic-induced representations. The first applications are the derivation of quantum mechanical theories with exceptional spectrum-generating superalgebras, like F(4) and G(3). The nonassociativity manifests itself in the moduli space of the theory as emergent coupling constants. It is argued that the "quasi-nonassociative" framework, which is based on a double role played by the octonionic structure constants, can offer a consistent dynamical formulation for the octonionic M-theory and, in condensed matter, to engineer materials with induced nonassociative features.

- **Yuri Manin**
  Title: *Quantum theta functions and signal analysis*
  Abstract: Representations of the celebrated Heisenberg commutation relations and their exponentiated versions form the starting point for a number of basic constructions, both in mathematics and mathematical physics (geometric quantization, quantum tori, classical and quantum theta functions) and signal analysis (Gabor analysis).
  In this talk I explain how Heisenberg relations bridge the noncommutative geometry and signal analysis.
  After providing a brief comparative dictionary of the two languages, I will show e.g. that the Janssen representation of Gabor frames with generalized Gaussians as Gabor atoms yields in a natural way quantum theta functions, and that the Rieffel scalar product and associativity relations underlie both the functional equations for quantum thetas and the Fundamental Identity of Gabor analysis.

- **Rita Fioresi**
  Title: *Unitarizable Harish-Chandra representations of real supergroups and Hermitian Symmetric Superspaces*
  Abstract: In 1955 Harish-Chandra constructed infinitesimally and globally representations of a real semisimple Lie algebra and the corresponding Lie groups. The global modules were realized as the spaces of sections of holomorphic line bundles on the associated hermitian symmetric space. He also provided conditions to ensure the unitarizability of such modules. We want to prove an analogue of all of these results in the super category. This is a joint work with C. Carmeli and V.S. Varadarajan.

- **Ruben Mkrtchyan**
  Title: *On a universal quantum dimension*
  Abstract: Universal, in Vogel's sense, formulae for (quantum) dimensions of irreps of simple Lie algebras, and their applications will be presented.

- **Alexei Kotov**
  Title: *Q-manifolds and gauge theories*
  Abstract: It will be shown how Q-manifolds with additional structures can be used in gauge theories of certain type.

- **Dimitry Leites**
  Title: *Supersymmetrized Schroedinger and KdV operators*
  Abstract: It was known since early 1900s that the product of two solutions of the stationary Schroedinger (aka Sturm—Liouville) equation

$$L_2(f) = 0, \text{ where } \ L_2 = \frac{d^2}{dx^2} + F(x),$$

  is a solution of the stationary KdV equation

$$L_3(g) = 0, \text{ where } L_3 = \frac{d^3}{dx^3} + \frac{d}{dx}F(x) + F(x)\frac{d}{dx}.$$

  In late 1970s A.A.Kirillov-père explained this fact with the help of supersymmetry, having considered elements of the coadjoint modules of the Virasoro algebra and Neveu–Schwarz superalgebra. I will show that in so doing Kirillov missed odd parameters — the very tsimes of supersymmetry.
  Since then, simple stringy superalgebras acting on superstrings of dimension $1|N$ were conjecturally classified, and their central extensions were classified: there are 12 of them. Therefore, there can be 12 super analogs of Schroedinger and KdV operators (in reality, there are less).
  I tried to solve the very first problem related to these operators that springs to mind: the eigenvalue one. Interestingly, it is hardly meaningful for $N$ odd, as I will show.

- **José Miguel Figueroa-O'Farrill**
  Title: *Lie superalgebra deformations and supersymmetry*
  Abstract: I will review the construction of a Lie superalgebra associated to any solution of the d=11 supergravity field equations and show that it is a filtered deformation of a graded subalgebra of the Poincaré superalgebra. Such deformations are governed by generalised Spencer cohomology and I will report on some calculations of the relevant Spencer cohomology groups in various dimensions. This results in PDEs for spinor fields having the property that their solutions generate a Lie superalgebra.

- **Valentin Ovsienko**
  Title: *q-deformation of real numbers*
  Abstract: The notion of q-deformation is a version of quantization; a number of q-deformations is known in algebra, geometry and combinatorics. However, q-deformations of numbers are well understood (since Euler and Gauss) only for integers. I will talk of a recent attempt, with Sophie Morier-Genoud, to develop a theory of q-deformed real (and rational) numbers. We associate a formal power series with integer coefficients to a positive real number; for negative real numbers we obtain certain Laurent series. I will give a combinatorial interpretation of the coefficients and some properties of the developed notion, such as the total positivity. The construction is similar to that of Gaussian q-binomial coefficients, the Pascal triangle being replaced by the Farey graph; it is related to the cluster algebras and "quantum Teichmüller space".

- **Ivan Penkov**
  Title: *Flag supermanifolds and Bott-Borel-Weil Theorem*
  Abstract: In the first part of the talk, I will explain a proof of the Bott-Borel-Weil Theorem for typical line bundles on flag supermanifolds. This is old work but I feel that it is not sufficiently well known. In the second part, I will speculate about similarities between this picture and newer work on ind-varieties of generalized flags, with the aim of introducing the yet inexistent field of ind-supergeometry.

- **Andrey Krutov**
  Title: *Nichols algebras and irreducible quantum flag manifolds*
  Abstract: In this talk we will discuss Nichols algebras naturally arising in the noncommutative Kähler geometry of the irreducible quantum flag manifolds. In particular, we present sufficient conditions allowing us to associate to any quantum principal bundle over a quantum homogeneous space a Yetter–Drinfeld module structure on the cotangent space of the base calculus. As a motivating example, this allows us to show that the holomorphic and anti-holomorphic Heckenberger–Kolb calculi of the quantum Grassmannians could be expressed as Nichols algebras.
  Joint work with Réamonn Ó Buachalla (Bruxelles) and Karen Strung (Prague).

- **Jian Qiu**
  Title: *Toric generalised Kahler manifolds and elliptic functions*
  Abstract: In this talk I shall sketch a construction of generalised Kahler 4-manifold for toric fano manifolds. It has been shown that certain type of generalised Kahler structures can be encoded in terms of a Morita equivalence of two holomorphic Poisson structures. We combine this with a construction due to Hitchin of bi-hermitian 4 manifolds in the toric setting, where, thanks to the action angle coordinates, one can be very explicit. We show that one obtains a generalised Kahler potential expressed in terms of Weistrass elliptic functions and also discuss possible application of this result. This is joint work with F.Bonechi and M.Tarlini.

- **Alexander Voronov**
  Title: *The moduli space of genus 0 super Riemann surfaces with Ramond punctures*
  Abstract: A construction of the $(n–3|n/2–2)$-dimensional supermoduli space of super Riemann surfaces of genus 0 with $n$ Ramond punctures is presented. This is a joint work with my graduate student Nadia Ott.

- **Richard Kerner**
  Title: *The $\mathbb{Z}_3-$graded extension of the Poincaré algebra*
  Abstract: A $\mathbb{Z}_3$ symmetric generalization of the Dirac equation was proposed in several years ago, and its properties and solutions discussed recently. The generalized Dirac operator acts on "coloured" spinors composed out of six Pauli spinors, describing three colours and particle-antiparticle degrees of freedom characterizing a single quark state, thus combining $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3$ symmetries of 12-component generalized wave functions.
  The $\mathbb{Z}_3-$graded generalized Lorentz algebra and its spinorial representation were introduced in a recent paper with J. Lukierski (Physics Letters, March 2019), leading to the appearance of extra $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3$ symmetries, probably englobing the symmetries of isospin, flavors and families.
  The present talk shows the construction of $\mathbb{Z}_3-$graded extension of the Poincaré algebra. It turns out that such a generalization requires introduction of extended 12-dimensional Minkowskian space-time containing the usual 4-dimensional space-time as a subspace, and two other mutually conjugate "replicas" with complex-valued vectors and metric tensors. Representation in terms of differential operators and generalized Casimir operators are introduced and their symmetry properties are briefly discussed.

- **Ted Voronov**
  Title: *Quantum thick morphisms, homotopy algebras and spinor representation*
  Abstract: "Classical" thick morphisms between manifolds or supermanifolds (also called microformal morphisms) are generalization of smooth maps — though which are not usual maps, rather relations between the corresponding cotangent bundles equipped with some extra data. They were discovered for the purpose of constructing L-infinity morphisms of higher (homotopy) brackets, when manifolds in question have an S-infinity ("homotopy Schouten") or P-infinity ("homotopy Poisson") structure. The key feature of thick morphisms is that they induce NONLINEAR, in general, pullbacks on functions. (This nonlinearity is exactly the feature making them useful for homotopy brackets purposes.) It was found that there is a "quantum version" of thick morphisms in the form of formal Fourier integral operators of special type. They also introduce L-infinity morphisms for a "quantum version" of S-infinity algebras (generated by BV-type operators). Remarkably, such "quantum pullbacks" can be also seen as a generalization of spinor representation in the sense of Berezin-Neretin, as it became clear recently — through the same sort "intertwining relation" that connects them with homotopy algebras. In the talk, I will try to explain all these connections.
  (See arXiv:1409.6475,arXiv:1411.6720,arXiv:1903.02884, also arXiv:1506.02417, arXiv:1512.04163, arXiv:1710.04335; and arXiv:1909.00290 , the latter joint with Hovhannes Khudaverdian.)

- **Ekaterina Shemyakova**
  Title: *Construction of super Plücker embedding in the general case: towards the notion of super cluster algebras*
  Abstract: The needs of the actively developing area of Cluster Algebras have drawn our attention to a problem which has remained unclarified/unsolved since the early years of Supergeometry: a superanalog for the Plücker embedding, which can be seen as a key motivating example for the much sought-after notion of "super cluster algebras". There has been active work towards definition of super cluster algebras (Ovsienko, Ovsienko-Shapiro, and Li-Mixco- Ransingh-Srivastava), but the notion is still a mystery.
  In the talk, we present our construction of the "super Plücker embedding" in the general case of the Grassmannian of $r|s$-planes in $n|m$-space.
  Only a very special case was considered before in the literature, namely, of $2|0$-planes in $4|1$-space (Cervantes-Fioresi-Lledó). We will show that the naive algebraic construction of exterior powers (with sign adjustments) can go through for the Grassmannian $G_{r|0}(n|m)$, i.e. the case of completely even planes in the superspace. For the general case of $r|s$-planes, we propose the following solution: our super Plücker map takes the Grassmann supermanifold $G_{r|s}(V)$ to a "weighted projective space" $P\left(\Lambda^{r|s}(V) \oplus \Lambda^{s|r}(\Pi V)\right)$ with weights $+1, -1$. Here $\Lambda^{r|s}(V)$ denotes the $r|s$ th exterior power of a superspace $V$ and $\Pi$ is the parity reversion functor. We identify the super analog of Plücker coordinates and show that our map is an embedding. We investigate a super analog of the Plücker relations. We obtain them for arbitrary $r|0$ and $n|m$. Also, we consider another type of relations due to H. Khudaverdian and show that they are equivalent to (super) Plücker relations for $r|s = 2|0$ (this is new even in the classical case), but in general are only a consequence of the Plücker relations. We also discuss possible application to super cluster algebras (which are, as mentioned above, only partly known at present).
  (Based on https://arxiv.org/abs/1906.12011. This is joint work with Th. Voronov.)

# Posters

- **Olena Atlasiuk**
  Title: *On Linear Boundary-Value Problems for the Differential Systems in Sobolev Spaces*

- **Kowshik Bettadapura**
  Title: *Variations of Splitting Type*

- **Andrew James Bruce & Steven Duplij**
  Title: *The Double-Graded Quantum Superplane*

- **Igor Khavkine**
  Title: *Initial data for closed conformal Killing-Yano 2-forms*

- **Krechko Viktoriia**
  Title: *On solutions of evolution equations describing the propagation of correlations in quantum systems in a mean field approximation*

- **Antonio Miti**
  Title: *Homotopy co-momentum maps in Multisymplectic geometry*

- **Ivan Penkov**
  Title: *integrable* $\mathfrak{sl}(\infty)-$*modules and category* $\mathcal{O}$ *for* $\mathfrak{gl}(m|n)$

- **Urs Schreiber**
  Title: *Twisted cohomotopy implies M-theory anomaly cancellation*

- **Vit Tuček**
  Title: *BGG sequences of differential operators*

- **Elizaveta Vishnyakova**
  Title: *TBA*

- **Marko Živković**
  Title: *Graph Complexes*