

## Schedule

All talks will take place in Maison du Savoir 2.400.

### Thursday 14th

10:00 - 11:00	Hovhannes Khudaverdian	“Thick morphisms and higher Koszul brackets”
11:00 - 11:30	Coffee break	
11:30 - 12:30	Janusz Grabowski	“Splitting theorem for colored supermanifolds”
12:30 - 15:00	Lunch	
15:00 - 16:00	Rita Fioresi	“Admissible systems and Harish-Chandra representations for semisimple real Lie supergroups”
16:00 - 16:30	Coffee break	
16:30 - 17:30	Norbert Poncin	“Higher supergeometry revisited”
18:30 -	Workshop dinner	Restaurant Beeftro à Esch-belval

### Friday 15th

10:00 - 11:00	Steven Duplij	“Developing new supermanifolds by revitalizing old ideas”
11:00 - 11:30	Coffee break	
11:30 - 12:30	Zoran Škoda	“Symmetry objects for geometries based on monoidal categories”
12:30 - 15:00	Lunch	
15:00 - 16:00	Florian Hanish	“Berezin integration on super loop space and applications to index theory”
16:00 - 16:30	Coffee break	
16:30 - 17:30	Andrew Bruce	“On a $\mathbb{Z}_2^n$ -graded version of Minkowski superspace”

## Abstracts

**Andrew Bruce** (University of Luxembourg)

### **On a $\mathbb{Z}_2^n$ -graded version of Minkowski superspace**

We show one possible way of extending the notion of Minkowski superspace to the setting of  $\mathbb{Z}_2^n$ -geometry. The resulting  $\mathbb{Z}_2^n$ -manifold formally resembles  $N$ -extended superspace (with central charges) but there are subtle differences due to the exotic nature of the grading employed.

**Steven Duplij** (University of Münster)

### **Developing new supermanifolds by revitalizing old ideas**

First, we apply the regularity concept for supermanifolds and obtain so called obstructed (or regular) supermanifolds (semi-supermanifolds), generalizing von Neumann regularity to  $n$ -regularity. In the patch definition, the transition functions become noninvertible in some sense and satisfy special cocycle conditions described by semi-commutative diagrams. The obstruction, which measures the distinction from the standard supermanifold, is proportional to the difference of some idempotent self-maps from identical morphisms. The same procedure of regularization is applied for categories and functors. Second, we introduce another noninvertible analog of superconformal transformations which arise from the alternative possibility of supermatrix reduction in the simplest one-grade case. They are always noninvertible, having nilpotent Berezinian and twist parity of the tangent space together with the so called mixed cocycle condition. For  $N = 2$  the role of superjacobian is played by permanent, also the remarkable formula connected Berezinian, permanent and determinant is obtained. Third, we consider some aspects of ternary supersymmetry. Fourth, we introduce the ring of polyadic integer numbers, which leads to unusual polyadic fields. The ways of generalization for the above constructions to the multigraded supersymmetry is outlined.

**Rita Fioresi** (University of Bologna)

### **Admissible systems and Harish-Chandra representations for semisimple real Lie supergroups**

Real forms of classical Lie superalgebras are in one to one correspondence with Cartan automorphisms and Cartan decompositions. We introduce the notion of admissible root system with respect to a given Cartan automorphism and we show that the condition of admissibility is equivalent to the existence of infinite dimensional holomorphic representations of the real Lie supergroups underlying the given real form. We briefly discuss generalization to Lie superalgebras infinite dimensional and to Kostant root systems.

**Janusz Grabowski** (Institute of Mathematics, Polish Academy of Sciences)

### **Splitting theorem for colored supermanifolds**

The classical Batchelor-Gawedzki theorem says that any smooth supermanifold is (non-canonically) diffeomorphic to the ‘superization’  $\Pi E$  of a vector bundle  $E$ . It is also known that this result fails in the complex analytic category. Hence, it is natural to ask whether an analogous statement goes through in the category of colored supermanifolds ( $\mathbb{Z}_2^n$ -supermanifolds) with its local model made of formal power series with  $\mathbb{Z}_2^n$ -gradation and  $\mathbb{Z}_2^n$ -commutation rules. We show that any smooth colored supermanifold is (non-canonically) diffeomorphic to the ‘superization’  $\Pi E$  of an  $n$ -fold vector bundle  $E$ . The latter can be chosen split.

**Florian Hanisch** (University of Potsdam)

### **Berezin integration on super loop space and applications to index theory**

The super loop space associated to an ordinary (spin) manifold is the space of “maps” from the super circle  $S^{1|1}$  to  $M$ . We will briefly describe its infinite-dimensional supermanifold structure and show that the resulting algebra functions is closely related to the algebra of differential forms on ordinary loop space. In particular, it is possible to relate the action of  $S^{1|1}$  on superfunctions to the equivariant differential on forms. Based on the Wiener integral, it is possible to define an analogue of Berezin integration. Similar to the finite-dimensional case, it turns out that the odd part of this integral may be expressed by an infinite-dimensional Pfaffian, the latter being defined using zeta-regularization. If time allows, we will eventually describe how these concepts can be used to obtain “path integral-type” proofs of index theorems by integrating Bismut-Chern characters. The argument indicates that supersymmetry enters these calculations in a crucial way.

This talk is based on joint work (partly in progress) with Matthias Ludwig

**Hovhannes Khudaverdian** (University of Manchester)

**Thick morphisms and higher Koszul brackets**

It is a classical result in Poisson geometry that the cotangent bundle of a Poisson manifold has the structure of a Lie algebroid. One of the manifestations of this structure is the “Koszul bracket” of differential forms. There is a natural homomorphism from the resulting differential Lie superalgebra into the superalgebra of multivector fields with respect to the canonical Schouten bracket and the Lichnerowicz differential. In the talk, we shall present a homotopy analog of the above results. When an ordinary Poisson structure is replaced by a homotopy one, instead of a single Koszul bracket there arises an infinite sequence of “higher Koszul brackets” introducing an  $L_\infty$ -algebra structure in the space of differential forms (Khudaverdian-Voronov, 2008, <http://arxiv.org/abs/0808.3406>). We shall show how to use thick morphisms of supermanifolds to construct a non-linear transformation, which is an  $L_\infty$ -morphism, from this  $L_\infty$ -algebra of differential forms to the Lie superalgebra of multivector fields with the canonical Schouten bracket. (A thick morphism of manifolds is a new notion recently introduced, see e.g. <http://arxiv.org/abs/1411.6720v5>.)

(Based on joint work with Th. Voronov.)

**Norbert Poncin** (University of Luxembourg)

**Higher supergeometry revisited**

The aim of the talk is to present a generalization of superalgebra and supergeometry to  $\mathbb{Z}_2^n$ -gradings,  $\mathbb{Z}_2^n = \mathbb{Z}_2 \times \cdots \times \mathbb{Z}_2$ ,  $n > 1$ . The corresponding sign rule is not given by the product of the parities, but by the scalar product of the involved  $\mathbb{Z}_2^n$ -degrees. This  $\mathbb{Z}_2^n$ -supergeometry exhibits interesting differences with classical supergeometry, provides a sharpened viewpoint, and has better categorical properties. Further, it is closely related to Clifford calculus: Clifford algebras have numerous applications in Physics, but the use of  $\mathbb{Z}_2^n$ -gradings has never been investigated. In particular, the  $\mathbb{Z}_2^n$ -Berezinian determinant and the corresponding integration theory will be discussed.

**Zoran Škoda** (University of Zagreb)

**Symmetry objects for geometries based on monoidal categories**

As the basic organizing principle of supersymmetry, as well as of its  $\mathbb{Z}_2^N$  refinement, one often emphasizes that all algebraic constructions live in an appropriate symmetric monoidal category. In the context of monoidal categories one studies the symmetry objects like supergroups and their anyonic and braided analogues. Using actions of these symmetry objects, one comes to important examples of geometries, the superhomogeneous and some quantum homogeneous spaces. After presenting some generalities on symmetries in the context of monoidal categories, I will focus on attempts at combining the infinitesimal symmetries or differential operators with the coordinate superalgebra of the space into the analogues of groupoids as novel symmetries in this context. Main examples comprise super-Hopf algebroids. I will also explain their relevance in the deformation quantization.



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