Interactions and Applications of Homotopical Algebra and Geometry

November 14-16, 2022, University of Luxembourg.

Talks

- **Nils Baas**
  Title: *Higher structures – a novel approach.*
  Abstract: In the talk I will motivate and introduce the concept of hyperstructures – a framework for higher structures. I will discuss extensions of field theory and motivate the introduction of neural field theories.

- **Wojciech Chachólski**
  Title: *Homological algebra and persistence.*
  Abstract: My presentation is based on an article coauthored with A. Jin and F. Tombari (arXiv:2112.12209).
  I will explain why upper semilattices play an important role in TDA and present a construction called realization that shares a lot of properties that are desirable in TDA with upper semilattices. For instance I will show that the homological dimension (the length of the minimal free resolution) and Betti numbers of tame vector space valued functors indexed by both realizations and upper semilattices can be calculated using Koszul complexes. Consequently, calculating these invariants for functors indexed by both types of posets can be done directly, avoiding constructing explicit resolutions.

- **Benoit Fresse**
  Title: *Calculus of embeddings of manifolds and (decorated) graph complexes.*
  Abstract: I propose to talk about recent research with Victor Turchin and Thomas Willwacher on the study of the rational homotopy of embedding spaces of manifolds through operads. The main outcome of this research, which I propose to explain in the talk, is a combinatorial description, in terms of (decorated) graph complexes, of the space of embeddings of a manifold (possibly with several connected components of different dimensions) into the Euclidean space $\mathbb{R}^n$. In related subjects, some results of our research give descriptions of deloopings of the spaces of embeddings with compact support of a Euclidean space $\mathbb{R}^m$ into $\mathbb{R}^n$ (the spaces of long knots in the case $m = 1$), as well as descriptions of spaces of rational homotopy automorphisms associated to operads.

- **Julien Grivaux**
  Title: *Deformation theory and quantized cycles.*

Abstract: In this talk, we will review some geometrical constructions attached to a closed embedding of schemes (or complex submanifolds). The first result is due to Huybrechts and Thomas, refining previous work of Lieblich and Lowen, and describes the first order deformation theory of a perfect complex of sheaves on the subscheme. It turns out that the existence of such deformation is controlled by a cohomology class that can be explicitly computed. A bit later, Arinkin and Caldararu provided a surprising link with the theory of derived intersections. We will present an extension of these results (with new and more conceptual proofs), and explain how this circle of ideas provides an interesting field of investigation.

- **Vladimir Hinich**
  Title: *Day convolution for enriched infinity categories.*
  Abstract: In (conventional) category theory Yoneda embedding $Y : \mathcal{C} \to P(\mathcal{C})$ enjoys certain universal properties:

  1. If $\mathcal{C}$ is monoidal or symmetric monoidal, $P(\mathcal{C})$ inherits this structure and Y is automatically (symmetric) monoidal.
  2. Any functor $f : \mathcal{C} \to \mathcal{D}$ into a category with colimits uniquely factors through Y.

  In this talk we present a version of these results for enriched infinity categories.

- **Andrey Lazarev**
  Title: *Homotopy theory of algebras and coalgebras.*
  Abstract: Differential graded algebras and differential graded coalgebras are ubiquitous in algebra, geometry and homotopy theory. The categories that they form have an intrinsic homotopy theory of their own. We explain that these categories, from a certain 'infinity-categorical' perspective, are equivalent. This, despite the fact that as ordinary categories, algebras and coalgebras cannot be more different.

- **Ieke Moerdijk**
  Title: *Profinite infinity-operads*
  Abstract: In this talk, based on joint work with Thomas Blom, I will describe a profinite completion of (simplicial or topological) operads, as a left Quillen functor from the model category of ∞-operads to a new model category of profinite ∞-operads. The construction is based on a notion of lean ∞-operad, and a characterization in terms of homotopical finiteness properties will be given of those ∞-operads that are weakly equivalent to lean ones. As an application, we will discuss the derived automorphism group of some such completed topological operads occurring in the work of Boavida-Horel-Robertson.

- **Joost Nuiten**
  Title: *Derived deformation theory in positive characteristic.*
  Abstract: Ever since the work of Kodaira and Spencer, homotopical methods have found fruitful applications in deformation theory. At the basis of many of these applications lies a principle emphasized by Deligne: the infinitesimal deformations of an algebro-geometric object over a field of characteristic zero are controlled by a differential graded Lie algebra, which one can study using tools from (operadic) homotopical algebra.

Recent work of Brantner and Mathew provides a variant of this principle in positive characteristic. In this talk, I will give an overview of this work and discuss the homotopical algebra that can be used to study the "partition Lie algebras" that control deformation problems in characteristic p. Based on joint work with L. Brantner and R. Campos.

- **Christian Saemann**
  Title: *EL_∞-algebras and adjusted connections on higher principal bundles*
  Abstract: I will review the construction of higher principal bundles with connection, focusing on the adjustment necessary for non-fake-flat connections. This adjustment motivates a generalization of strong homotopy Lie algebras to a weaker form which we call EL_∞-algebras. These algebras have the potential to answer many questions, from the algebraic structures underlying generalized geometry and the gauge algebras arising in gauged supergravities to the integration of Leibniz algebras.

- **Hisham Sati**
  Title: *M-theory and homotopy algebras*
  Abstract: I will provide an overview of the connections between M-theory on the one hand and rational homotopy theory and homotopy Lie theory on the other. At the heart of these connections is Hypothesis H, the observation that the fields in M-theory are governed by maps to the 4-sphere. Many consequences of mathematical and physical interest arise from this.

- **Claudia Scheimbauer**
  Title: *A universal property of the higher category of spans and finite gauge theory as an extended TFT*
  Abstract: I will explain how to generalize Harpaz' universal property of the $(\infty, 1)$-category of spans to the higher category thereof. The crucial property is "m-semiadditivity", which generalizes usual semiadditivity of categories. Combining this with the finite path integral construction of Freed-Hopkins-Lurie-Teleman this yields finite gauge theory as a fully extended TFT. This is joint work in progress with Tashi Walde.

- **Alexander Schenkel**
  Title: *Derived algebraic geometry in mathematical physics*
  Abstract: Derived algebraic geometry is a powerful geometric framework which plays an increasingly important role both in the foundations of algebraic geometry and in mathematical physics. It introduces a refined concept of 'space', the so-called derived stacks, that is capable to describe correctly geometric situations that are problematic in traditional approaches, such as non-transversal intersections and quotients by non-free group actions. In this talk I will give a very basic introduction to derived algebraic geometry, focusing in particular on its more concrete and computational aspects. I will then illustrate the potential of this framework for new developments in mathematical physics by studying two applications:

  1. The derived critical locus of a function $f : [X/G] \to \mathbb{K}$ on a quotient stack, and
  2. the quantization of a derived cotangent stack $\mathbb{T}^*[X/G]$ over a quotient stack.

This talk is based on joint works with Benini and Safronov [arXiv:2104.14886] and Benini and Pridham [arXiv:2201.10225].

- **Martina Scolamiero**
  Title: *Stable Homological Invariants for data*
  Abstract: One of the aims of Topological Data Analysis is to produce invariants for topological spaces arising from data (e.g a finite metric space). An important property in this context is that such invariants should be stable with respect to small perturbations of the data. A strategy to achieve stability is to define a continuos map between appropriate metric spaces representing the data and its representation. In the first part of this talk I will give an overview of a method developed by the TDA group at KTH to define distances between algebraic representations of data (persistence modules) and use them to define a stable invariant given any numerical invariant of the persistence module. Special focus will be placed on the computability of such stable invariants. In the second part of the talk, I will discuss relative homological algebra invariants promising in the context of TDA, recently proposed jointly with W.Chacholski, A.Guidolin, I.Ren and F.Tombari.

- **Sergey Shadrin**
  Title: *From quantization of hydrodynamic integrable systems to Givental Grothendieck Teichmüller group.*
  Abstract: We will sketch a construction of quantization of integrable hierarchies of PDE's of hydrodynamic type (think of the KdV hierarchy) via the method of Buryak-Rossi. It involves the quantum cohomological field theories, and we'll discuss the definition and properties of the homotopy version of this concept. We'll show that there emerges a natural action by the Kontsevich graph complex amplified by the so-called tautological classes on the moduli space of curves (it is a further development of an idea of Merkulov-Willwacher). The cohomology of this complex contain on the same footing the Lie algebras of the Givental group of symmetries of cohomological field theories (thus extended to the homotopy world) and the Grothendieck-Techmüller group.

- **Bruno Vallette**
  Title: *The properadic calculus.*
  Abstract: Over the last 30 years, a conceptual and efficient operadic calculus has been extensively developed. It now provides us with complete toolbox to treat the algebraic and homotopical properties of algebras over an operad: deformation complex, gauge group, infinimorphisms, homotopy transfer theorem, twisting procedure, etc. In this talk, I will report on the recent developments of the extension of this calculus to the level of properads which encode types of bialgebras, including motivating examples of applications like double Poisson gebras, pre-Calabi–Yau algebras, and the topological recursion, for instance.

Posters

- **Julian Bruggemann:** *On Merge Trees and Discrete Morse Functions on Paths and Trees*
- **Luigi Caputi:** *Controlled objects as a symmetric monoidal functor*
- **Leonardo Ferrari:** *Geometric cobordisms of rational homology 3-spheres*
- **A. Ali Hakeem:** *On homotopical property for topological left almost semigroups.*
- **Boughazi Hichem:** *The first GJMS conformal invariant*
- **Ruben Louis:** *Symmetries of Singular Foliations through Universal Lie ∞-algebroids*
- **Luuk Stehouwer:** *Spin-statistics in two-dimensional topological field theory*