Expressibility of digraph homomorphisms in the logic LFP+Rank

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One of the fundamental problems in finite model theory is the quest for the logic which captures polynomial time on finite (di)graphs. From the algebraic point of view, an interesting restriction of this problem asks whether there is a logic $L$ strong enough to capture, given a finite digraph $G$, the class $\neg \text{HOM}(G)$ of all finite digraphs not homomorphic to $G$ and such that the truth of $L$-sentences on finite digraphs can be decided in polynomial time. In 2009, Atserias, Bulatov, and Dawar showed that the LFP+C cannot capture the homomorphism problem on digraphs, where C is the counting operator. Recently, with Bulin, Jackson, and Niven, we refined the original method of Feder and Vardi of translating the constraint satisfaction problem for general relational structures to digraphs in such a way that it preserves the algebraic reasons for polynomial time solvability. In this talk, we present a very recent result, obtained with F. McInerney, which shows that, under the aforementioned transformation, if $\neg \text{HOM}(A)$ is definable by a LFP+Rank sentence, then $\neg \text{HOM}(D_A)$ is definable in the same logic, where $D_A$ is the digraph obtained from the relational template $A$. In conclusion, we discuss some related conjectures.