What amounts to algebra and what is due to analysis in the invention and uses of complex numbers, from the fundamental theorem of algebra to Heisenberg relations

JEAN DHOMBRES

École des hautes études en sciences sociales, France

According to a structural point of view, the fact that there are only three Banach fields, and the complex field among the three, proves the fundamental algebraic character of complex numbers. But an opposite point of view recalls that the terminology for the fundamental theorem of algebra has been Ironically coined by Gauss to show that there is essentially the need for some result from analysis. The purpose of my talk is not to discuss a priori these two opposite points of view, but to look at the ways proofs were invented for complex numbers from the moment, around 1750, when the complex exponential was understood, and so I would like to study in the same line the requirement for complex Hilbert spaces and the proofs of Heisenberg relations in quantum mechanics.