On quantifiers on pocrims

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Monadic MV-algebras (MMV-algebras, in short) were introduced and studied by J. Rutledge [7] as an algebraic model of the predicate calculus of the Łukasiewicz infinite valued logic in which only a single individual variable occurs. MMV-algebras were also studied as polyadic algebras by D. Schwarz [8], [9]. Recently, the theory of MMV-algebras has been developed in [1], [2] and [3].

The results have been recently extended in [6] for GMV-algebras (pseudo-MV-algebras), which form a non-commutative generalization of MV-algebras.

Recall that monadic, polyadic and cylindric algebras, as algebraic structures corresponding to classical predicate logic, have been investigated by Halmos in 60’s and by Henkin, Monk and Tarski. Similar algebraic structures have been considered for various logics in [4] and [5].

The aim of our talk is to built up the theory monadic operators in a more general setting, namely for bounded pocrims. Bounded pocrims form a large class of algebras containing as proper subclasses the class of BL-algebras (an algebraic semantics of Hájek’s BL-logic) as well as the class of Heyting algebras (algebras of intuitionistic logic).

We show that for so-called normal pocrims, i.e. those satisfying the identity

$$\neg(x \odot y) = \neg x \odot \neg y$$

(where $\neg x = x \rightarrow 0$), there is a mutual correspondence between existential and universal quantifiers. Further, the correspondence of existencial quantifiers with the $m$-relatively complete substructures will be discussed.

REFERENCES