On monoidal characterization of closed maps

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There exist two different characterizations of the same class of continuous maps between topological spaces. Given a continuous map \( f : X \to Y \), on the one hand, \( f \) is said to be proper provided that the product map \( f \times 1_Z : X \times Z \to Y \times Z \) is closed for every topological space \( Z \). On the other hand, \( f \) is said to be perfect provided that it is closed, and the fibre \( f^{-1}(y) \) is compact for every \( y \in Y \). N. Bourbaki showed the equivalence of these two concepts, and emphasized the importance of proper maps in general topology. The result of N. Bourbaki generalizes the famous Kuratowski-Mrówka theorem, the latter stating that for every topological space \( X \), the unique map \( X \to 1 \) is perfect if and only if it is proper. Recently, M. M. Clementino and W. Tholen [1] extended the above two results to monoidal topology [2], which is a new approach to categorical topology, based in monads and quantales. Their machinery, however, relies on a monoidal analogue of the notion of closed map, which requires the involved quantales to be constructively completely distributive. Following the quantale independent definition of proper map of [1], in this talk, we propose an alternative concept of closed map, which, firstly, is not dependent on the involved quantales; secondly, coincides with the notion of M. M. Clementino and W. Tholen in their proposed framework of complete distributivity; and, thirdly, makes the relationships between monoidal versions of proper and closed maps easier to follow (every proper map is then trivially closed). We also show that the analogues of the Kuratowski-Mrówka and Bourbaki theorems are valid in the new setting as well.

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References