

ABSTRACT

A step towards Lie's dream

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The origins of Lie theory are well known: Galois theory had clarified the relationship between the solutions of polynomial equations and their symmetries. Lie had attended lectures by Sylow on Galois theory and came up with the idea to develop a similar theory for differential equations and their symmetries which he and coworkers then successfully built. At a certain stage, they noticed that “transformations groups” with finite-dimensional Lie algebra was a very tractable area. This resulted in a brilliant and complete theory, that of Lie groups, but the connection with the origins gets somewhat lost.

The idea of a Galois theory for differential equations prompted as well what has come to be known as differential Galois theory. We will present a general approach that encompasses ordinary Galois extensions (of commutative rings), differential Galois theory, and principal bundles (in differential geometry and algebraic geometry). The new notion that we introduce for that purpose is that of *principal comorphism of Lie-Rinehart algebras*.