Topic for Bachelor thesis

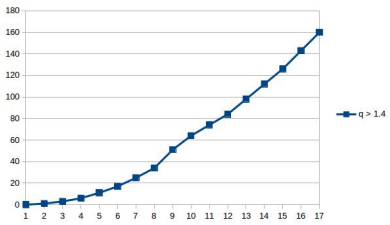
**Supervisor:** Alexander D. Rahm

**Title:** *Experimental tests on the abc conjecture*

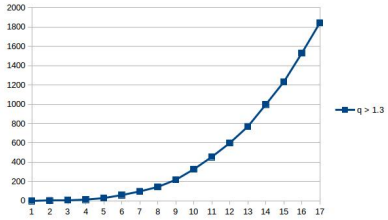
**Short Description:** The abc conjecture is about triples of positive integers, $a, b$ and $c$ that are relatively prime and satisfy $a + b = c$. The quality $q(a, b, c)$ of the triple $(a, b, c)$ is defined as $q(a, b, c) = \log(c)/\log(\text{rad}(a \cdot b \cdot c))$, where the radical $\text{rad}(a \cdot b \cdot c)$ is the product of the distinct prime factors of the product $a \cdot b \cdot c$.

*abc conjecture:* For every positive real number $\varepsilon$, there exist only finitely many triples $(a, b, c)$ of coprime positive integers with $a + b = c$ such that $q(a, b, c) > 1 + \varepsilon$. The amazingly important consequences of the abc conjecture are described on wikipedia [1].

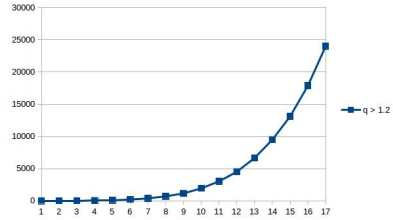
The ABC@home non-profit network computing project with its more than 7,300 active participants from 114 countries did calculate all abc-triples of at most 18 digits. Concerning the data that it has collected, the abc conjecture implies that the following graphs of the number of abc triples of quality $q$ above the respectively fixed threshold (plotted over the number of digits they admit minus 1) are each one bounded by some constant:



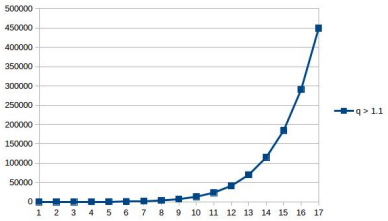

(a) $q > 1.4$



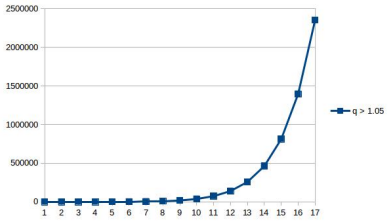

(b) $q > 1.3$




(c) $q > 1.2$




(a) $q > 1.1$




(b) $q > 1.05$

On Prof. Bart de Smit's ABC triples page [2], it can be seen that the growth of the first curve ($q > 1.4$) stops when continuing to 19 and 20 digits (which still does not prove that there is a constant bounding the curve). But for the abc conjecture to be true, also the curves for $q > 1.01, q > 1.001, 1.0001, \ldots$ need to be bounded by a constant each.

An infinite sequence of abc triples of quality above a fixed threshold greater than 1 would disprove the abc conjecture. *Question.* How close can one get to counterexamples to the abc-Conjecture?

**Goal:** We do not aim at finding an actual counterexample. Rather, we want to use the database collected by the ABC@home grid computing project in order to see how close we can get to a counterexample, constructing infinite sequences using a certain strategy of the project's supervisor, which has been checked back not to be known already by Prof. Bart de Smit, who was in charge of the ABC@home network.

**Examples of Literature:**

[1] Wikipedia, *The ABC conjecture,* `https://en.wikipedia.org/wiki/Abc_conjecture`

[2] Bart de Smit, *ABC triples page*, `https://www.math.leidenuniv.nl/~desmit/abc/?set=2`

**Requirements:**

Courses in Algebra and Linear Algebra, ability to use a programming language.

**Language:** English, French or German.