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Andersen, Jorgen, Aarhus, Denmark

The Hitchin connection, Toeplitz operators and TQFT

The Witten-Reshetikhin-Turaev Topological Quantum Field Theory in particular provides us with the so call quantum representations of mapping class groups. The geometric construction of these involves geometric quantization of moduli spaces, which produced a holomorphic vector bundle over Teichmller space. This bundle supports a projectively flat connection constructed by algebraic geometric techniques by Hitchin. We will present a differential geometric construction of this connection in a generalized setting. Furthermore its relation to Toeplitz operators will be discussed. In fact we will see that the parallel transport of this connection is a Toeplitz operator, hence manifestly placing Toeplitz operators within the study of TQFT's. We will further give applications of this, like the asymptotic faithfulness of these quantum representations and asymptotic expansions of the quantum invariants. Finally we will also discuss the their application in our proof that the mapping class groups do not have Kazhdan's property T.

Baier, Thomas, Lisbon, Portugal

Geometric quantization of families of degenerating complex structure

Using a weak formulation (on distributions) of the equations of covariant constancy as a unified setting for Geometric Quantization in both positive (i.e. complex) and non-negative (real or mixed) polarizations, we study some aspects of the behaviour of the associated quantum bundle as the polarization degenerates for two families of examples, toric manifolds (based on the preprint arXiv:0806.0606) and abelian varieties.

Bondal, Alexey, Aberdeen, UK

Orthogonal decomposition of Lie algebras and a problem of quantum information

We will outline the problem and known results on decomposing Lie algebra sl(n) into an orthogonal sum of Cartan subalgerbas. We will describe the relation of this problem to generalized Hecke and Temperley-Lieb algebras, and to harmonic analysis on graphs. We explain why the problem have recently attracted a lot of attention by math physicists: relation to quantum information transmission and to discrete versions of the Feynman path integral. A new 4-dimensional family of orthogonal pairs in sl(6) will be presented. This is a joint work with I. Jdanovsky.

Charles, Laurent, Paris, France

Quantization of polygon space

Moduli spaces of polygons have been studied since the nineties for their topological and symplectic properties. Under generic assumptions these are symplectic manifolds with natural global action angle coordinates. Applying geometric quantization to the polygon spaces, one obtains invariant subspaces of tensor product of several irreducible representation of SU(2). These quantum spaces admit natural sets of commuting observables. I will explain in which sense these operators quantize the action coordinates and how we can use them to prove the asymptotics of the 6j-symbols.

Dai, Xianzhe, Santa Barbara, US

The asymptotic expansion of the Bergman kernel

The Bergman kernel in the context of several complex variables has long been an important subject. Its analogue for complex projective manifolds is studied by Tian and Zelditch, among others, establishing the diagonal asymptotic expansion for high powers of an ample line bundle. Moreover, the coefficients in the asymptotic expansion encode geometric information of the underlying complex projective manifolds. This asymptotic expansion later found many applications. We will discuss the Bergman kernel of the $spin_c$ Dirac operator associated to high powers of an ample line bundle in the general context of symplectic manifolds and orbifolds and establish the full off-diagonal asymptotic expansion and Agmon estimate. This is joint work with K. Liu and X. Ma.

Driver, Bruce, San Diego, USA

Holomorphic functions and subelliptic heat kernels over Lie group

(Based on joint work with Laurent Saloff-Coste and Leonard Gross) A Hermitian form q on the dual space, $\mathfrak{g}^*$, of a Lie algebra, $\mathfrak{g}$, of a Lie group, G, determines a Laplacian, Δ, on G. Assuming Hörmander's condition for hypoellipticity, the subelliptic heat semigroup, $e^{t\Delta/4}$, is given by convolution by a C^∞ probability density ρ_t. Analogous to earlier work in the strongly elliptic case, we are able to show that if G is complex, connected, and simply connected then the Taylor expansion defines a unitary map from the space of holomorphic functions in $L^2(G, \rho_t)$ onto (a subspace of) the dual of the universal enveloping algebra in the norm induced

Englis, Mirek, Prague, Tschech Republic

Toeplitz quantization on real symmetric domains

An analogue of the Berezin-Toeplitz star product, familiar from deformation quantization, is studied in the setting of real bounded symmetric domains. The analogue turns out to be a certain invariant operator, which one might call star restriction, from functions on the complexification of the domain into functions on the domain itself. In particular, we establish the usual (i.e. semiclassical) asymptotic expansion of this star restriction, and describe real-variable analogues of several other results. This is a joint work with Harald Upmeier (Marburg).

Fujita, Hajime, Tokyo, Japan

Torus fibrations and localization of index

In this talk I will talk about a localization of index of a Dirac type operator. We make use of a structure of torus fibration, but the mechanism of the localization does not rely on any group action. Two examples of applications are:
(1) For a prequantizable closed symplectic manifold equipped with a Lagrangian fibration the Riemann-Roch number is localized at Bohr-Sommerfeld fibers and singular fibers.
(2) For a Hamiltonian torus action on a prequantizable closed symplectic manifold the Riemann-Roch number is localized at the inverse image of the integral lattice points for the moment map.
To show the localization we introduce a deformation of a Dirac type operator on a manifold equipped with a fiber bundle structure which satisfies a kind of acyclic condition. The deformation allows an interpretation as an adiabatic limit or an infinite dimensional analogue of Witten deformation. (Joint work with Mikio Furuta and Takahiko Yoshida.)

Gorodentsev, Alexey, Moscow, Russia

Functorial A infinity coproduct of combinatorial simplicial chains that induces itself under barycentric subdivision

Consider two simplicial chain complexes: one associated with a combinatorial simplicial complex Δ having a finite set of vertexes and another one associated with the barycentric subdivision of Δ ("combinatorial" means that each simplex in Δ is completely determinated by its vertexes). I'll show that in zero characteristic there exists unique (up to an obvious rescalling) strong homotopy retractiction between these two chain complexes that is functorial in Δ (in particular, equivariant w.r.t. the permutations of vertexes) and write precise formulas for this retraction data. For any functorial A_∞-coproduct of combinatorial simplicial chains this retraction produces another such a product via usual "sum over trees" inducing procedure. I'll show that there is a unique functorial A_∞-coproduct that induces itself under such barycentric subdivisions. For 1-dimensional standard simplex $[0, 1]$ this barycentrically self-inducing coproduct is given by the Todda power series. In general case it should be closely connected with Campbell - Hausdorf formula and non-commutative symmetric functions but today I don't k now a nice closed formula for it.

Huebschmann, Johannes, Lille, France

Line bundles on moduli and related spaces

Let G be a Lie goup, let M and N be smooth connected G-manifolds, let $f\colon M \to N$ be a smooth G-map, and let P_f denote the *fiber* of f. Given a closed and equivariantly closed relative 2-form for f with integral periods, we construct the principal G-circle bundles with connection on P_f having the given relative 2-form as curvature. Given a compact Lie group K, a biinvariant Riemannian metric on K, and a closed Riemann surface Σ of genus ℓ, when we apply the construction to the particular case where f is the familiar relator map from $K^{2\ell}$ to K, which sends the 2ℓ-tuple $(a_1, b_1, \ldots, a_\ell, b_\ell)$ of elements a_j, b_j of K to $\prod[a_j, b_j]$, we obtain the principal K-circle bundles on the associated extended moduli spaces which, via reduction, then pass to the corresponding line bundles on possibly twisted moduli spaces of representations of $\pi_1(\Sigma)$ in K, in particular, on moduli spaces of semistable holomorphic vector bundles or, more precisely, on a smooth open stratum when the moduli space is not smooth. The construction also yields an alternative geometric object, distinct from the familiar gerbe construction, representing the fundamental class in the third integral cohomology group of K or, equivalenlently, the first Pontrjagin class of the classifying space of K.

Kaledin, Dmitry, Moscow, Russia

Cotangent bundles of complex Grassmanians and representation theory

Consider the cotangent bundles $T^*Gr(m,n)$ of complex Grassmanians, all of them at once. These are smooth complex algebraic varieties equipped with a holomorphic symplectic form. About ten years ago Nakajima has shown that the equivariant K-theory of these varieties realizes in a natural way finite-dimensional representation of the affine quantum group sl_2 (this is the simplest case of his general result about arbitrary affine quantum groups and the corresponding "quiver varieties", of which $T^*Gr(m,n)$ are the simplest example). Results of Kashiwara, Lusztig, Vasserot, Nakajima and others show that consequently, these K-theory spaces admit a certain "canonical basis" with very good and restrictive properties. However, no direct geometric construction of such a basis is known at present.

I will show how my recent results on an algebraic description of the derived categories $D^b(X)$ of smooth holomorphically symplectic varieties allow for such a geometric construction, why the conditions on the canonical basis are very natural from the geometric point of view, and what they entail for the geometry of $T^*Gr(m,n)$ and the structure of the derived category $D^b(T^*Gr(m,n))$.

Karabegov, Alexander, Abilene, USA

Formal symplectic groupoids with separation of variables and Bergman idempotent

We describe in detail a construction of the formal symplectic groupoid corresponding to a deformation quantization with separation of variables. Then we use this construction to give a formal model of a Bergman projection operator.

Kori, Tosiaki, Tokyo, Japan

Geometric pre-quantization of SU(N) flat connections on 3-manifolds

Let M be a compact connected and oriented three dimensional riemannian manifold. Let $G = SU(N)$, $N \geq 2$. Let $\mathcal{A}_0^\flat(M)$ be the space of flat connections A that satifies $CS_{(3)}(A) = -\frac{1}{24\pi^3}\int_M tr\, A^3 = 0$. Then $\mathcal{A}_0^\flat(M)$ is endowed with a pre-symplectic structure ω defined by

$$\omega_A(a,b) = \frac{1}{24\pi^3}\int_M tr[\,(ab-ba)\,A\,], \quad a,b \in T_A\mathcal{A}_0^\flat(M). \tag{1}$$

It is observed that $\mathcal{A}_0^\flat(M)$ is the space of boundary restrictions of flat connections on a four-manifold that cobord M.

Now suppose M is the boundary of a four-manifold X, and suppose that X is a submanifold of a connected, simply connected and compact four-manifold that cobord a five-manifold. Then there exists a hermitian line bundle with connection $\mathcal{L}^X$ over $\mathcal{A}_0^\flat(M)$, of which the curvature is given by $-i\omega$. This is the Chern-Simons pre-quantum line bundle. For the group $G = SU(2)$ we need a special investigation because of the fact $\pi_4(SU(2)) = \mathbf{Z}_2$, while $\pi_4(SU(N)) = 0$ for $N \geq 3$. The group of gauge transformations on M acts on $\mathcal{A}_0^\flat(M)$ by an infinitesimally symplectic way. When X is the 4-dimensional disc we show that this action is lifted to the pre-quantum line bundle $\mathcal{L}^X$ by its abelian extension. The geometric description of the latter is related to the 4-dimensional Wess-Zumino-Witten model.

Khudaverdian, Hovhannes, Manchester, UK

Differential forms in an odd symplectic geometry and Pfaffians

The square root of the Berezinian (superdeterminant) of an odd symplectic transformation is a polynomial in the matrix entries. We make this observation and study its consequences for odd symplectic geometry. In particular, we study relations of the above fact with homological interpretation (suggested by P. Severa) of odd Laplacian acting on semidensities. We draw attention to the fact that the de Rham complex on a manifold M naturally admits an action of the supergroup of all canonical transformations of the odd symplectic supermanifold ΠT^*M. The talk is mainly based on my joint work with Th. Th. Voronov.

Ma, Xiaonan, Paris, France

Bergman kernel and geometric quantization

Mano, Toshiyuki, Kyoto, Japan

The Riemann-Wirtinger integral and its application

The Riemann-Wirtinger integral is a function defined by a definite integral whose integrand is a power product of theta functions and the exponential function. It was found as a special solution to the isomonodromic deformation equation of linear differential equations on an elliptic curve. It might be regarded as an elliptic analogue of well-known Gauss' hypergeometric function. We give it a formulation based on twisted homology and cohomology theory on an elliptic curve. We shall also mention to some related topics.

Marinescu, George, Cologne, Germany:

Toeplitz operators on complex and symplectic manifolds

We study the Berezin-Toeplitz quantization making use of the full off-diagonal asymptotic expansion of the Bergman kernel. We give also a characterization of Toeplitz operators in terms of their asymptotic expansion. The semi-classical limit properties of the Berezin-Toeplitz quantization for non-compact manifolds and orbifolds are also established. This is a joint work with Xiaonan Ma

Moriyama, Takayuki, Kyoto, Japan

Deformations of transverse Calabi-Yau structures on foliated manifolds

We develop a deformation theory of transverse structures given by closed forms on foliated manifolds. We apply Goto's deformation theory to transverse structures on foliated manifolds and show that a deformation space of the transverse structures is smooth under a cohomological condition. As an application, we obtain unobstructed deformations of transverse Calabi-Yau structures on foliated manifolds.

Natanzon, Sergey, Moscow, Russia

Topological string theory

The Topological String Theory is a topological approximation of a String Theory. The String Theory is a modern variant of physical "Theory of Everything". In the topological approximation we assume that a particle isn't point but an one-dimensional object. Then a trajectory of the particle is a surface. It the topological approximation we consider that probability of the trajectory depend only on topological type of the surface and parameters of born/death of the particle. This assumption gives to simple system of axioms, that independently appear in different fields of mathematics from abstract algebra to integrable systems.

Nohara, Yuichi, Nagoya, Japan

Toric degenerations of Gelfand-Cetlin systems and potential functions

It is well known that a polarized toric variety is related to a moment polytope in two different ways, monomial basis and the moment map. Also for flag manifolds, certain polytopes, called Gelfand-Cetlin polytopes, appear in similar ways: the Gelfand-Cetlin basis, a basis of an irreducible representation; and the Gelfand-Cetlin system, a completely integrable system. Furthermore the flag manifold admits a degeneration into a toric variety corresponding to the Gelfand-Cetlin polytope. Kogan and Miller proved that the Gelfand-Cetlin basis can be deformed into monomial basis on the toric variety under the degeneration.
We show that the Gelfand-Cetlin system can be deformed into a moment map on the toric variety. We also apply the result to disk counting and calculate the potential function for a Lagrangian torus fiber of the Gelfand-Cetlin system. This is a joint work with T. Nishinou and K. Ueda.

Osipov, Denis, Moscow, Russia

Formal groups arising from formal punctured ribbons

Formal groups arising from formal punctured ribbons. Abstract. Formal punctured ribbon is a generalization of punctured formal neighbourhood of a divisor on algebraic surface. We investigate Picard functor of a formal punctured ribbon. We prove that under some conditions this functor is representable by a formal group scheme, which can be decomposed into the product of formal Brauer group and some infinite-dimensional variety. It is expected that solutions of Parshin's generalization of KP-hierarchy will be flows on these Picard schemes. (The talk is based on joint works with H. Kurke and A. Zheglov.)

Paoletti, Robert, Milano, Italy

Local asymptotics for Toeplitz operators

Toeplitz operators are the compression of pseudodifferential operators with a Szego kernel, that is, whith the orthogonal projector onto a Hardy space; as such, they imply the existence of a Toeplitz structure, which is a generalization of the notion of strictly pseudoconvex domain. A particularly important class of Toeplitz structure is provided by the unit circle bundles of positive (ample) line bundles. A motivation for the study of Toeplitz operators stems from geometric quantization, as they are natural candidates for the quantization of a classical observable represented by a smooth function on the base manifold. We shall focus on the local asymptotics of Toeplitz operators in this setting, by first discussing local intepretations of the Weyl law and the trace formula, and then, in the equivariant case, by studying how eigenfunctions lying in certain families of spectral bands asymptotically concentrate in the expected region of phase space.

Shadrin, Sergey, Amsterdam

Lax operator algebras and integrable systems

BCOV theory associates some action to the space of polyvector fields on Calabi-Yau varieties. The perturbative expansion of this action is conjectured (and proved in some cases) to coincide up to a change of variables with the Gromov-Witten potential of the mirror dual variety
This approach was formalized mathematically by Barannikov-Kontsevich. They associated a Frobenius structure to an arbitrary Batalin-Vilkovisky algebra satisfying the Hodge condition. Later on, Losev, me, and Shneiberg have found an alternative interpretation of the same Frobenius structure that allowed us to construct a mirror analogue of the full descendant Gromov-Witten potential. In particular, by some miracle, our construction has appeared to satisfy all possible universal relations that reflect topology of the moduli space of curves. that reflect topology of the moduli space of curves.
In this talk I am going to explain an explicit construction of the underlying cohomological field teory in terms of divisors on the moduli space of curves.

This gives a new and a much more simple interpretation of BCOV theory and, in particular, all complicated theorems about universal relations in BCOV theory turn out to be almost obvious. The core of this new construction is a special case of the Givental group action on the space of Frobenius manifolds and cohomological field theories.

Schottenloher, Martin, Munich, Germany

The geometric phase in QED

We report on a program aiming to establish a mathematically rigourous description of QED, initiated by Fierz, Scharf and Mickelsson, among others. The program is geometric in nature and it is based on a formulation within infinite dimensional bundles. These Fock space bundles have fibers which are isomorphic to a conventional version of fermionic Fock space for a fixed infinite dimensional separable Hilbert space $\mathcal{H}$, and they have as base spaces suitable moduli spaces of potentials on Minkowski space or the Grassmannian of $\mathcal{H}$ modeled on the Hilbert space of Hilbert-Schmidt operators $\mathcal{H} \to \mathcal{H}$. As a first step one has to determine the geometric phase in a completely natural manner because physical entities like the polarization current depend explicitly on it. In order to achieve this the geometric approach has been used to construct the time evolution modulo its phase for smooth and compactly supported electromagnetic potentials (even with nonzero magnetic part) in the following paper:

Deckert, Dürr, Merkl, Schottenloher: *Time evolution of the external field problem in QED*. arXiv:0906.0046

Sheinman, Oleg, Moscow, Russia

Lax operator algebras and integrable systems

The (extended) spaces of Tyurin data generalize moduli spaces of semistable holomorphic vector bundles on Riemann surfaces. Every point of such space is assigned with an almost graded Lie algebra called Lax operator algebra (such Lie algebras generalize loop algebras). The elements of that Lie algebra are parameterized by cotangent vectors at the point. By Lax operator we mean the mapping sending a point of the cotangent bundle to an element of the Lax operator algebra. We consider certain dynamical systems given by Lax equations on the cotangent bundle to a space of Tyurin parameters. We formulate that these dynamical systems are Hamiltonian with respect to the Krichever-Phong symplectic structure, and integrable. Examples include Hitchin systems, Calogero-Moser and some more systems. Finally, we formulate the problem of quantization of those systems.

Shoikhet, Boris, Luxembourg

Bialgebras and n-monoidal categories

We construct a 2-fold monoidal category structure on the category of tetramodules over a bialgebra A. We explain how to deduce from this claim that the Gerstenhaber-Schack cohomology of any Hopf algebra is a 3-algebra

Talalaev, Dmitry, Moscow, Russia

Bethe Ansatz and Isomonodromic transformations

We study symmetries of the Bethe equations for the Gaudin model appeared naturally in the framework of the geometric Langlands correspondence under the name of Hecke operators and under the name of Schlesinger transformations in the theory of isomonodromic deformations, and particularly in the theory of Painleve transcendents.

Tate, Tatsuya, Nagoya, Japan

An asymptotic Euler-Maclaurin formula for Delzant polytopes

We mean, by the name asymptotic Euler-Maclaurin formula for Delzant polytopes, formulae on asymptotic expansion of the Riemann sums over lattice polytopes. The Riemann sums over lattice polytopes have applications to geometry and spectral analysis on toric varieties. An asymptotic Euler-Maclaurin formula is obtained due to Guillemin-Sternberg. Thus, problem is to find an effective formula for each term of the asymptotics. In this talk, after giving a brief review of the history, we give a new asymptotic formula for the Riemann sums over Delzant polytopes, which is quite similar to the exact local Euler-Maclaurin formula obtained by Berline-Vergne.

Treschev, Dmitry, Moscow, Russia

Non-commutative structures and operators on Hilbert spaces

We present a method to associate operators on a Hilbert space with elements of a version of the Heisenberg algebra. The aim is to join algebraic and functional-analytitic approach to the objects of quantum mechanics.

Tyurin, Nikolai, Dubna, Russia

Non toric lagrangian fibrations of toric (and non toric) Fano varieties

D.Auroux presented recently an example of non toric lagrangian fibration of the projective plane, very important in his programme of special lagrangian fibrations of Fano varieties. His construction can be generalized for toric Fano varieties and even non toric ones (which we call pseudo toric). Thus a number of methods in Geometric Quantization and Mirror Symmetry suitable for toric manifolds can be applied in much more broad context.

Upmeier, Harald, Marburg, Germany

Generalized Fock spaces on Jordan varieties

We present a new realization of unipotent representation spaces of semisimple Lie groups, which generalize the well-known Fock space of entire functions under the metaplectic representation of the symplectic group. Our construction works for all Lie groupsG of hermitian type, which correspond to holomorphic automorphisms of hermitian symmetric domain s G/K of non-compact type. Using the well-known Matsuki duality of orbits in flag manifolds, we construct a family of Hilbert spaces over the base manifold G/K, whose fiber at the origin is the generalized Fock space, and the other fibers are obtained by an (infinite-dimensional) connection on G/K which is projectively flat (curvature is a scalar). This connection yields the (projective) unitary representation of G although the underlying Jordan variety is not a G-orbit for a holomorphic action. In a sense,G/K becomes the moduli space of complex structures on a Jordan variety of lower dimension, which serves as a phase space. In the talk we outline the general construction, including exceptional Lie groups, and describe the relation to the standard Fock spaces. The parallel transport "Bogolyubov" transformations for the new connection are also described via their integral kernels. Unlike the well known Bergman spaces of holomorphic functions, the generalized Fock spaces make sense for infinite-dimensional groups, since the symmetric domain G/Kdoes not need a measure but only a connection. Finally, we point out that the construction may hold in much greater generality to also include non-holomorphic representations.

Yoshida, Takahiko, Japan

Torus fibrations and localization of index -application to locally toric Lagrangian fibrations

This is a supplemental talk to Hajime Fujita's lecture on a localization of index of a Dirac type operator. As an application of the localization we show the equality between the Riemann-Roch number and the number of Bohr-Sommerfeld fibers for 4-dimensional closed locally toric Lagrangian fibration.

Wendland, Katrin, Augsburg, Germany

Asymptotic analysis for unitary conformal field theories

We discuss limiting processes for conformal field theories and degeneration phenomena, which allow us to associate geometric interpretations to limits of conformal field theories by applying techniques from noncommutative geometry. The construction mimics semiclassical limits, but it is performed in an intrinsic manner, without preassuming the knowledge of a target space geometry.