School Geoquant 2019

The focus on the **2019 Geoquant school** will be on the quantization of moduli spaces of bundles, Higgs bundles, integrable systems, Kähler metric, conformal field theory, and more.

Those topics are clearly in the center of the current interest in the field. We are happy that we could find leading experts in the field which are willing to deliver introductory lectures. The following lectures are foreseen:

- Akito Futaki (University of Tokyo, Tokyo; Tsinghua University, Beijing), *GIT stability, constant scalar curvature Kähler metrics and deformation quantizations*

  A fundamental problem in Kähler geometry is finding a Kähler metric of constant scalar curvature on quantizable Kähler manifolds (polarized manifolds). A necessary and sufficient condition is conjectured to be the so-called K-stability in geometric invariant theory. The special case for Fano manifolds has been confirmed by Chen-Donaldson-Sun and Tian, in which case the existence of Kähler-Einstein metrics is proved. Recently La Fuente-Gravy and myself found a similar link between the existence of closed Fedosov star product on quantizable Kähler manifolds and a new notion of K-stability condition expressed in terms of Cahen-Gutt moment map. In these lectures we formulate a cohomology formula of the relevant invariant expressing the Mumford weight. The plan of the lectures are as follows:

  1. Basic ideas and recent developments on Yau-Tian-Donaldson conjecture on the existence of Kähler metrics of constant scalar curvature.

  2. Deformation quantization, the star products by Fedosov and Berezin-Toeplitz, balanced embeddings of quantizable Kähler manifolds.

  3. Constant Cahen-Gutt momentum and a notion of K-stability condition.

  4. Outline of the proof of the existence of Kähler-Einstein metric on Fano manifolds

- Nan-Kuo Ho (National Tsing Hua University, Hsinchu), *On the moduli space of flat bundles*

  This is an introductory lecture on the moduli space of flat bundles. We will consider moduli spaces where the structure group is a compact Lie group, or a complex Lie group. We will look at some topological results and some symplectic results of the moduli space. If timepermits, we will also look at the moduli space of flat bundles with an involution.

- Igor Krichever (Skoltech Center for Advanced Studies, Moscow), *Algebraic-geometrical integration theory of soliton equations and its applications*

  The course tries to give a self-contained introduction to the theory of soliton equations with an emphasis on its applications to algebraic-geometry. Topics include:

  1. General features of the soliton systems. Lax representation. Zero-curvature equations. Integrals of motion. Hierarchies of commuting flows. Discrete and finite-dimensional integrable systems.
  2. Algebraic-geometrical integration theory. Spectral curves. Baker-Akhiezer functions. Theta-functional formulae.
  3. Hamiltonian theory of soliton equations.
  4. Commuting differential operators and holomorphic vector bundles on the spectral curve. Hitchin-type systems.
  5. Characterization of the Jacobians (Riemann-Schottky problem) and Prym varieties via soliton equations.
  6. Perturbation theory of soliton equations and its applications.

- Motohico Mulase(University of California, Davis), *Quantization of Higgs bundles through the Gaiotto correspondence*

  The Physicist Gaiotto conjectured a remarkable quantization mechanism of Hitchin spectral curves that constructs a one-parameter family of opers from a given spectral curve. The conjecture was solved in a collaboration of Dumitrescu, Fredrickson, Kydonakis, Mazzeo, Naitzke and me (Journal of Differential Geometry 2019). Recently Collier and Wentworth have pointed out that the same analysis method can be applied to quantizing more general Higgs bundles. I will explain Gaiotto's conjecture, its solution, and current developments from the algebraic geometry point of view. If time permitting, I will also discuss variation of Hodge structures and D-modules from this new point of view. Lectures based on my on-going collaboration with Dumitrescu, A. Jacob, and C. Jacob.

- Richard Wentworth (University of Maryland, College Park) *Determinant line bundles, Deligne pairings, and quantization*

  Outline:

  1. 1. Determinant bundles and Deligne pairings. Quillen metrics. Deligne isometry. Examples.
  2. Bosonization formulas. Scalar and bc-theories. Spin 1/2 bosonization, insertion theorem, and the Bismut-Lebeau result. Explicit formulas in terms of theta functions. Relation with Faltings' invariant.
  3. Gluing of determinants. Path integral approach. BFK formula and generalizations to Dolbeault laplacians. Determination of explicit constants.
  4. Deligne pairings for families of flat connections. Cappell-Miller torsion. Hyperholomorphic line bundles.

Each of these courses are planned for 2-4 sessions each lasting one hour. In addition to them we will allow short presentations (e.g. posters, talks by the participants). As they have been proven very effective in former schools organized in the frame of the GEOQUANT activity, regular discussion sessions are foreseen. There students can ask the lecturers additional questions in a less formal way, demand for more background information or even present by themselves certain aspects.

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