Workshop on higher structures
13–16 December 2016

Venue: Room B27 (main building)

Thesis defense

Xiongwei Cai will defend his thesis in Salle des Conseils at 10h30.

Title: Cohomologies and derived brackets of Leibniz algebras
Abstract: We work on the structure of Leibniz algebras and develop cohomology theories for them.

- We introduce standard cohomology and naive cohomology for a Leibniz algebra. We discuss their properties and show that they are isomorphic. By similar methods, we prove a generalization of Ginot-Grutzmann's theorem on transitive Courant algebroids, which was conjectured by Stienon-Xu. The relation between standard complexes of a Leibniz algebra and its corresponding crossed product is also discussed.
- We observe a canonical 3-cochain in the standard complex of a Leibniz algebra. We construct a bracket on the subspace consisting of so-called representable cochains, and prove that the subspace becomes a graded Poisson algebra. Finally we show that for a fat Leibniz algebra, the Leibniz bracket can be represented as a derived bracket.
- Inspired by the notion of a Lie algebra action and the idea of generalized geometry, we introduce the notion of a generalized action of a Lie algebra $\mathfrak{g}$ on a smooth manifold M, to be a homomorphism of Leibniz algebras from $\mathfrak{g}$ to the generalized tangent bundle $TM + T^*M$. We define the interior product and Lie derivative so that the standard complex of $TM + T^*M$ becomes a $\mathfrak{g}$ differential algebra, then we discuss its equivariant cohomology. We also study the equivariant cohomology for a subcomplex of a Leibniz complex.

Program:

Tuesday, December 13

14:00-14:30	Welcome / Registration
14:30-15:30	Honglei Lang
15:30-16:00	Coffee break
16:00-17:00	Florian Schätz

Wednesday, December 14

10:00-11:00	Yaël Frégier
11:00-11:30	Coffee break
11:30-12:30	Camille Laurent-Gengoux
12:30-14:00	Lunch break
14:00-15:00	Martin Schlichenmaier
15:00-16:00	Zhangju Liu
16:00-16:30	Coffee break
16:30-17:30	Olivier Elchinger
19:30	Conference dinner at restaurant Le Bouquet Garni

Thursday, December 15

10:00-11:00	Boris Shoiket
11:00-11:30	Coffee break
11:30-12:30	Sergei Merkulov
12:30-14:00	Lunch break
14:00-15:00	Ping Xu
15:00-16:00	Mathieu Stienon
16:00-16:30	Coffee break
16:30-17:30	Mykola Matviichuk

Friday, December 16

09:45-10:45	Norbert Poncin
10:45-11:00	Coffee break
11:00-12:00	Xevi Guitart

Titles and Abstract:

- Olivier ELCHINGER (University of Luxembourg)

  Title: Harrison cohomology and commutative deformations

  Abstract: Harrison (co)homology can be defined via shuffle product or eulerian idempotents. Using convolution products, we can establish the equivalence of these definitions simply. This cohomology provides a framework for commutative formality: as shown by Kontsevich, (commutative) formality implies (commutative) deformation. In this setting, we compute Harrison cohomology for a certain class of hypersurfaces in order to study their deformations.

- Yaël FRÉGIER (Université d'Artois)

  Title: On some constructions of L_∞-algebras

  Abstract: Gerstenhaber and Schack have introduced a cohomology governing infinitesimal deformations of diagrams of algebras. It recovers Hochschild and Kodaira-Spencer theories in specific cases. However, more structure is needed to describe the formal deformations as a Maurer-Cartan equation. We will explain how one can build a L_∞-algebra governing such deformations in two ways: by resolution of operads or by derived brackets. Examples encompass some simultaneous deformation problems such as morphisms and algebras. The derived bracket approach can find applications in geometry, where the operadic approach does not apply.

- Xevi GUITART (Universitat de Barcelona)

  Title: Fields of definition of elliptic k-curves with CM and Sato-Tate groups of abelian surfaces

  Abstract: Let A be an abelian surface over a number field k that is isogenous over the algebraic closure of k to the square of an elliptic curve E. If E does not have CM, by results of Ribet and Elkies concerning fields of definition of elliptic k-curves, E is isogenous to a curve defined over a polyquadratic extension of k. We show that one can adapt Ribet's methods to study the field of definition of E up to isogeny also in the CM case, as long as k contains the field of CM. As an application of this analysis, we provide a number field over which abelian surfaces can be found realizing each of the 52 possible Sato–Tate groups of abelian surfaces. This is joint work with Francesc Fité.

- Honglei LANG (Max Planck Institute for Mathematics)

  Title: Strongly homotopy Lie algebras, homotopy Poisson manifolds and Courant algebroids

  Abstract: We study Maurer–Cartan elements on homotopy Poisson manifolds, which unify many twisted and homotopy structures. Using the fact that the dual of an L_∞-algebra is a homotopy Poisson manifold, we obtain a Courant algebroid from a 2-term L_∞-algebra $\mathfrak{g}$ via the degree 2 symplectic NQ-manifold $T^*[2]\mathfrak{g}^*[1]$. By integrating the Lie quasi-bialgebroid associated to the Courant algebroid, we obtain a Lie-quasi-Poisson groupoid from a 2-term L_∞-algebra, which is proposed to be the geometric structure on the dual of a Lie 2-algebra. These results also lead to a construction of a new 2-term L_∞-algebra from a given one, which could produce many interesting examples.

- Camille Laurent-Gengoux (University of Lorraine)

  Title: Finitely generated Lie-Reinhart algebroids

  Joint work with Lavau and Strobl.

- Zhangju Liu (Peking University)

  Title: Double Principal Bundles

  Abstract: We define double principal bundles, for which the frame bundle of a double vector bundle, double Lie groups and double homogeneous spaces are basic examples. It is shown that a double vector bundle can be realized as the associated bundle of its frame bundle. Also dual structures, gauge transformations and connections in a double principal bundle are investigated. This is a joint work with H.L. Lang and Y.P. Li.

- Mykola Matviichuk (University of Toronto)

  Title: On the deformation theory of Dirac structures

  Abstract: In 2011 Hitchin proved that for a holomorphic Poisson manifold X, each 1st order deformation in $H^1(X,T)$ coming from the Poisson tensor is unobstructed. Later, Fiorenza and Manetti reinterpreted the Hitchin's result as a formality statement for the Koszul dgla on the space of de Rham forms of X. We consider the deformation problem of Dirac structures, of which Poisson structures are particular examples. Given a Dirac structure L, one can construct a dgla on the space of L-forms governing the deformation theory of L. To do this, one has to make a choice of a transverse Dirac, and the resulting dgla does depend on the choice. However, we prove that different choices of the transverse Dirac lead to L_∞ isomorphic dgla's. The proof uses the spinor approach to Dirac geometry and a version of Voronov derived bracket construction. We discuss applications to quasi-Poisson geometry, formality of Lie bialgebras and generalized Kaehler geometry. (This is a joint project with Marco Gualtieri and Geoffrey Scott)

- Sergei Merkulov (University of Luxembourg)

- Norbert Poncin (University of Luxembourg)

  Title: On horizontal and vertical categorification of Leibniz algebras

  Abstract: We start considering the horizontal categorification (oidification) of Leibniz algebras and define two subclasses of classical Leibniz algebroids: Loday algebroids and symmetric Leibniz algebroids. Whereas standard Leibniz algebroids carry only a left anchor, Loday algebroids are equipped with a standard left and a generalized right anchor, so that their brackets satisfy a differentiability condition on both arguments. Symmetric Leibniz algebroids are characterized by two weak differentiability conditions affecting both arguments and formulated in terms of the symmetrized Leibniz bracket. These two subclasses contain most of the Leibniz brackets that appear in the literature, but no one is included in the other. Loday algebroids admit a supergeometric interpretation. Such interpretations are known for Lie algebroids, homotopy Lie algebroids, and homotopy algebras over any quadratic Koszul operad P. Symmetric Leibniz algebroids are the underlying object of generalized Courant algebroids, a broader category that has free objects over anchored vector bundles. The construction of the free generalized Courant algebroid allows to show that not only derived brackets are Leibniz brackets, but that, conversely, symmetric Leibniz algebroid brackets can be (universally) represented by a derived bracket. Finally we pass to the vertical categorification (homotopyfication) of

P-algebras and investigate the categorical structure of homotopy P-algebras. Whereas the objects and morphisms of this category are well-understood, their homotopies are not. At least 5 candidates do exist. We explain that they are all equivalent, define higher homotopies and show that homotopy P-algebras form, not a 2-, but an infinity category. This holds in particular for homotopy Leibniz algebras. A concrete application of Getzler's integration technique for Lie infinity algebras allows to prove that the known (but quite mysterious) 2-categorical structure on 2-term homotopy Leibniz algebras is in fact the shadow of the infinity categorical structure on all homotopy Leibniz algebras.

- Florian Schätz (University of Luxembourg)

  Title: Deformations of pre-symplectic structures and generalized Koszul brackets

  Abstract: A pre-symplectic structure on a manifold is a closed two-form of constant rank. Pre-symplectic structures arise naturally in classical mechanics, for instance in the process of reduction. Moreover, they are closely related to coisotropic submanifolds and give rise to foliations. It is therefore an interesting problem to understand the space of all pre-symplectic structures of a given rank. I will describe a local parametrization of this space, as the set of Maurer-Cartan elements of an L_∞-algebra. This parametrization is best understood in terms of Dirac geometry. The talk is based on joint work with Marco Zambon (KU Leuven).

- Martin Schlichenmaier (University of Luxembourg)

  Title: N-point Virasoro algebras considered as Krichever–Novikov type algebras

  Abstract: We explain how the recently again discussed N-point Witt, Virasoro, and affine Lie algebras are genus zero examples of the multi-point versions of Krichever–Novikov type algebras as introduced and studied earlier by me. Using this more general point of view, useful structural insights and an easier access to calculations can be obtained.

- Boris Shoiket (Antwerp University)

  Title: Deligne conjecture for 1-monoidal categories

  Abstract: I will talk on my recent results on Deligne conjecture for monoidal abelian categories. Roughly speaking, I will present a machine constructing a B_∞ algebra on $X = \mathrm{RHom}_M(e, e)$ where M is an abelian monoidal category, and e is its unit. I will introduce a new concept of a graded Leinster monoid, a refined version of Leinster monoids, essential for the construction.

- Mathieu Stienon (Pennsylvania State University, USA)

- Ping Xu (Pennsylvania State University, USA)

  Title: Fedosov dg manifolds and Gerstenhaber algebras associated with Lie pairs

  Abstract: We study two cohomology groups, which serve as replacements for the spaces of "polyvector fields" and "polydifferential operators" on a pair (L, A) of Lie algebras (or more generally, Lie algebroids). In particular, we prove that both cohomology groups admit Gerstenhaber algebra structures. Our approach is based on the construction of a homological vector field Q on the graded manifold $L[1]+L/A$ and of a dg foliation (which we call Fedosov dg Lie algebroid) on the resulting dg manifold $(L[1] + L/A, Q)$.

- Sergei Merkulov (University of Luxembourg)

  Title: An explicit two step quantization of Poisson structures and Lie bialgebras

  Abstract: We develop a new approach to deformation quantizations of Lie bialgebras and Poisson structures which goes in two steps. In the first step one associates to any Poisson (resp. Lie bialgebra) structure a so called quantizable Poisson (resp. Lie bialgebra) structure. We show explicit transcendental formulae for this correspondence. In the second step one deformation quantizes a quantizable Poisson (resp. Lie bialgebra) structure. We show again explicit transcendental formulae for this second step correspondence. In the Poisson case the first step is the most non-trivial one and requires a choice of an associator while the second step quantization is essentially unique, it is independent of a choice of an associator and can be done by a trivial induction. The talk is based on a joint work with Thomas Willwacher.

- Mathieu Stienon (Pennsylvania State University, USA)

  Title: Kontsevich–Duflo theorem for Lie pairs

  Abstract: The Kontsevich–Duflo theorem asserts that, for any complex manifold X, the Hochschild–Kostant–Rosenberg map twisted by the square root of the Todd class of the tangent bundle of X is an isomorphism of associative algebras form the sheaf cohomology group $H^\bullet(X, \wedge T_X)$ to the Hochschild cohomology group $HH^\bullet(X)$. We will show that, beyond the sole complex manifolds, the Kontsevich–Duflo theorem extends to a very wide range of geometric situations describable in terms of Lie algebroids and including foliations and actions of Lie groups on smooth manifolds. A Lie pair (L, A) consists of a Lie algebroid L together with a Lie subalgebroid A. To each Lie pair are associated two Gerstenhaber algebras, which play roles similar to the spaces of polyvector fields and polydifferential operators. The Hochschild–Kostant–Rosenberg map twisted by the square root of the Todd class of the Lie pair yields an isomorphism between these two Gerstenhaber algebras.