Problems from mathematical contests

1. **Manhattan Mathematical Olympiad 2003**
   Prove that from any set of one hundred integers, one can choose either one number which is divisible by 100, or several numbers whose sum is divisible by 100.

2. **Japan 1997*** Prove that among any ten points located on a circle with diameter 5, there exist at least two at a distance less than 2 from each other.

3. **IMO 1959** Prove that the fraction $\dfrac{21n+4}{14n+3}$ is irreducible for every natural number n.

4. **IMO 1981***** Determine the maximum value of m^2+n^2, where m and n are integers in the range $1, 2, \ldots, 1981$ satisfying

$$(n^2 - mn - m^2)^2 = 1\,.$$

* You can start by

***Careful with the last problem, which contains non-school mathematics that we have not handled yet! The aim is that you look at its solution
https://artofproblemsolving.com/wiki/index.php?title=1981_IMO_Problems/Problem_3
and write in details the parts that you understand (you learn mathematical honesty and you learn working with black-boxes).