Logic

If and only if / Si et seulement si / Genau dann, wenn

Logic is important for all mathematics! In mathematics, true means ‘always true’. Something which is often true is declared to be false.

Negation: Negating an assertion exchanges true with false, and negating twice does nothing:

$$2+3 \text{ equals } 5 \text{ (true)} \rightsquigarrow 2+3 \text{ is not equal to } 5 \text{ (false)} \rightsquigarrow 2+3 \text{ equals } 5 \text{ (true)}$$

And, or: If you use the word AND to connect two assertions, they should both be true in order that the whole phrase is true:

$$2+3=5 \text{ (true) } \mathbf{and}\ 4+5=8 \text{ (false)} \qquad \text{is a false statement}$$

If you use the word OR to connect two assertions, at least one of them (possibly, both) should be true in order that the whole phrase is true:

$$2+3=5 \text{ (true) } \mathbf{or}\ 4+5=8 \text{ (false)} \qquad \text{is a true statement}$$

If you use the construction EITHER...OR to connect two assertions, then exactly one of them should be true (if both are true, or both are false, then the statement is false)

$$\mathbf{either}\ 2+3=5 \text{ (true) } \mathbf{or}\ 4+5=9 \text{ (true)} \qquad \text{is a false statement}$$

The negation exchanges *and* and *or*!

The negation of “I want pizza and pasta” is “I don’t want pizza or I don’t want pasta”

Maybe I want only pizza, maybe only pasta, maybe nothing at all...

The negation of “I want cinema or theater” is “I don’t want cinema and I don’t want theater”

I want neither of them!

Implications: The word **If** suggests an implication: ‘If you do this, I will do that.’ There is a premise (you do this) and a consequence (I do that). An implication, meant as a whole, is true *unless the premise is true but the consequence is false*. Your boss could tell you :

If you come late at work, you are fired.

Maybe there are other behaviours leading to your dismissal (you could get fired even if you are punctual) but the boss is not keeping his/her word only if you come late at work and he/she does not fire you.

Usually you cannot swap premise and consequence:

If you have 2 candies, then you have an even number of candies (true)

If you have even number of candies, then you have 2 candies (false)

Necessity: The construction **only if** suggests a necessary condition:

Only if you have an even number of candies you can have 2 candies (true)

Equivalence: Two conditions are equivalent means that one is necessary and sufficient for the other:

You have 2 candies, **if and only if** you have an even number of candies from 1 and 3 (true)

Contraposition (of an implication): The implication 'If A, then B' ($A \Rightarrow B$) gives you exactly the same information as 'If not B, then not A' ($notB \Rightarrow notA$).

If a triangle has 3 equal angles, then it has 2 equal angles

If a triangle does not have 2 equal angles, then it has not 3 equal angles

Transitivity of an implication: Knowing both implications 'If A, then B' and 'If B, then C' gives you automatically the implication 'If A, then C'. For example, 'If I am in Belval, I am in Luxembourg' and 'If I am in Luxembourg, I am in Europe' give automatically:
'If I am in Belval, I am in Europe'.

A **rule** is usually expressed by the words "Every", "All"...:

Every triangle has three sides. All triangles have three sides.

A rule is false even if there is only one single exception (counterexample). You cannot prove a rule just by looking at a few examples.

An **example** (or a counterexample) is usually expressed by the words "there is/are":

There is one mammal that can run at a speed of 30km/h.

There are mammals that can run at a speed of 30km/h.

In mathematics "there is one" means "there is at least one" (another phrasing is "one can find a").

The negation exchanges "All" and "There is":

The negation of "All mammals can move at a speed of 50km/h." is "There is at least one mammal that cannot move at a speed of 50km/h."

The negation of "Every mammal cannot reach a speed of 100km/h." is "There is at least one mammal that can reach the speed of 100km/h."

• "For all x and for all y" is the same as "For all y and for all x".

For every monkey and for every tree, the monkey can climb the tree.

For every tree and for every monkey, the monkey can climb the tree.

These are equivalent and mean: Every monkey can climb every tree (no exceptions for the monkeys, no exceptions for the trees).

• "There is an x and there is an y" is the same as "There is an y and there is an x".

There is a tree and there is a cat such that the cat can climb the tree.

There is a cat and there is a tree such that the cat can climb the tree.

Here it suffices to know that my cat can climb the tree in my garden. There are many possibilities: maybe one tree is too much for every cat, or there is a cat that can climb no tree, or every cat can climb some tree but not all, or every cat can climb every tree...

• "There is an y, such that for all x" could be stronger than "For all x there is an y".
The difference is that in the first case y must be the same for every x:

For every animal there is an appropriate food (true)

There is some food which is appropriate to every animal (false)

Problems around Logic

1. **Yes or No?** Associate correctly a wish with its negation (and explain your choice):

   Please, let's watch Star Wars or Star Trek!
   Please, let's watch Star Wars and Star Trek!

   Please, let us not watch Star Wars or let us not watch Star Trek!
   Please, let us not watch Star Wars and let us not watch Star Trek!

2. **Fishes and sharks** Which of the following implications convey the same information? Which among them are true?

   (a) Shark $\Rightarrow$ Fish
   (b) Fish $\Rightarrow$ Shark
   (c) Fish $\Leftrightarrow$ Shark
   (d) Not a fish $\Rightarrow$ Not a shark
   (e) Not a shark $\Rightarrow$ Not a fish
   (f) Not a fish $\Leftrightarrow$ Not a shark

3. **Zebra logic** Zebras are black&white. Old movies are black&white. So zebras are old movies! What went wrong in the reasoning?

4. **A zoo of possibilities** Order the following assertions from the strongest to the weakest (are any two assertions the same?):

   (a) At the zoo there is an animal that flies or swims
   (b) At the zoo there is an animal that flies or there is an animal that swims
   (c) At the zoo there is an animal that flies and swims
   (d) At the zoo there is an animal that flies and there is an animal that swims

5. **Darwin!** Darwin and Mendel studied plants, but we do not know which ones... Among the following assertions there are two that can be true at the same time and false at the same time. There are also two assertions that can be false at the same time but cannot be true at the same time. Which are they?

   (a) Darwin studied all plants that Mendel studied.
   (b) There is a plant that Mendel studied and that Darwin also studied, but Darwin did not study all plants that Mendel studied.
   (c) There is a plant that Mendel studied and that Darwin did not study.

6. **A matter of preferences** Which of the following assertions mean the same, which one is the strongest?

   (a) Every student liked every movie that was played at school.

   (b) Every movie played at school was liked by one of the students.

   (c) One of the students liked every movie played at schools.

   (d) Among the movies played at school, there was one liked by every student.

   (e) Every student liked one of the movies played a school.

ANY QUESTION? JUST ASK!