

PhD Away Days 2018

May 24 – 27, 2018, Heidelberg

Titles & Abstracts

Alexey KALUGIN «*Tannakian duality and applications*»

Abstract. Suppose that we have connected topological space X with the corresponding category of local systems $\text{Loc}(X)$ (locally constant sheaves of \mathbb{k} -vector spaces) on this space. Then choice of a point $x \in X$ in the topological space X determines the equivalence of this category with the category of finite dimensional \mathbb{k} -representations of the fundamental group $\pi_1(X, x)$ of the topological space X . Grothendieck and Deligne developed very beautiful construction which tells when given abelian category can be identified with the category of representations of some algebraic group. This idea appeared to be very seminal in the theory of mixed motives, geometric Langlands conjecture and other topics. We are going to discuss basic definitions and applications of this theory.

Andrea TAMBURELLI «*Harmonic maps and Teichmüller theory*»

Abstract. After a brief overview of the well-known theory of harmonic maps between open sets of \mathbb{R}^n , we will generalise this notion to general Riemannian manifolds and, in particular, hyperbolic surfaces. This will lead to a famous result by Michael Wolf which gives a parameterisation of Teichmüller space in terms of holomorphic quadratic differentials. We will conclude mentioning some generalisations of this theory to other geometric structures and open questions.

Daniel BERHANU «*A quick tour of the additive partition functions*»

Abstract. The aim of this concise presentation is to prove some recurrence relations for the restricted additive partition functions and to develop a closed formula which relates the Ramanujan's tau function with the unrestricted partition function.

Eduardo IBARGUENGOYTIA «*Descartes meets locally ringed spaces*»

Abstract. In this talk, I pretend to extract the algebraic properties hidden in the classical definition of a smooth manifold, in such a way that its definition as a locally ringed space appears naturally and allows generalizations in the more abstract and general algebraic geometric setting.

Emiliano TORTI «*Reciprocity laws and Galois representations.*»

Abstract. In this seminar, we will introduce the notion of reciprocity laws, starting from the quadratic one studied by Fermat, Euler and Gauss. We will focus on how the study of such laws helped and influenced the development of number theory. We will conclude by generalizing the notion of reciprocity laws in the modern language of Galois representations.

Guenda PALMIROTTA «*Visualizing the root system of Lie groups*»

Abstract. In 2007, worldwide in the media, we can read that the Atlas of Lie Groups and Representations members visualized the root system of the most exceptional Lie groups, namely the E_8 . The aim of this talk is to present in a simple way what the root systems of the classical and exceptional Lie groups are and how they look like.

Hanh VO «*Closed geodesic on pair of pants*»

Abstract. A Riemann surface of signature (g, n) is an oriented, connected surface of genus g with n boundary components, called boundary geodesics, which is equipped with a metric of constant curvature -1 . The length spectrum is the set, listed in ascending order, of lengths of closed geodesics of a Riemann surface. Almost every compact Riemann surface (i.e. a surface without boundary components) is determined by its length spectrum, up to isometry. Such surfaces can be decomposed into a succession of Riemann surfaces of signature $(0, 3)$, more commonly called a pair of pants, so it is important to study the length spectrum of a pair of pants. Moreover, the first elements of the length spectrum of a surface yield a lot of information about the surface itself, thus we are interested in the study of geodesics having a short length. In this talk, we define a parameter – the number of strings – and we classify geodesics using this parameter. We show that for a given number of strings, a minimal geodesic exists. We then give its length which depends on the boundary geodesics.

Jill ECKER «*The low-dimensional algebraic cohomology of the Virasoro algebra*»

Abstract. In this presentation, I am going to talk about the low-dimensional algebraic cohomology of the Virasoro algebra. I will start by giving some motivation and by indicating the main objectives. Next, I will introduce the Witt algebra, which is closely related to the Virasoro algebra. The Virasoro algebra is presented in a subsequent step. I will continue by presenting the Lie algebra cohomology and by giving an interpretation of the low-dimensional cohomology. Finally, I will present some results, namely that the third algebraic cohomology with values in the adjoint and the trivial module of the Virasoro algebra is one-dimensional.

Luca NOTARNICOLA «*The Pollard $p - 1$ algorithm, An Integer Factorization Algorithm*»

Abstract. Integer factorization has since always been an important challenge in mathematics and especially in number theory. With the advent of public key cryptography it is also of practical importance, because the security of some of these cryptosystems, such as the Rivest-Shamir-Adelman (RSA) system, depends on the hardness of factoring the public keys. In this talk we will introduce so called Pollard $p - 1$ factorization method, a simple algorithm which allows to

factor composite integers $N = pq$ where p and q are distinct prime numbers that satisfy special properties. Some ways of generalizing this algorithm to better functionality will be pointed out towards the end.

Mariagiulia DE MARIA «*Pell's equations*»

Abstract. Let d be a positive integer that is not a square. A diophantine equation of the form $x^2 - dy^2 = 1$ is called *Pell's equation*. In this talk, we will study integral positive solutions of these type of equations and we will see how they can be constructed in elementary ways, but we will also stress their connection to fundamental results of Number Theory such as the Dirichlet's unit theorem. In order to do so, we will recall the structure and properties of the ring $\mathbb{Z}[\sqrt{d}]$. At the end of the talk, we will discuss *generalized Pell's equations*, i.e. equations of the form $x^2 - dy^2 = n$ for d as above and n a non-zero integer, and their solutions.

Massimo NOTARNICOLA «*Recurrence and transience of the simple random walk on \mathbb{Z}^d* »

Abstract. Imagine a person is walking around randomly in a city that is arranged in a square grid. At each position, the person chooses one of the four possible directions independently of the past choices. Formally, that is how one can imagine the random walk in \mathbb{Z}^2 . An interesting question is then whether the person visits its starting point infinitely many times. One calls the random walk recurrent if this is the case and transient if not.

In this talk, we consider the simple random walk in \mathbb{Z}^d starting at the origin for any dimension $d \geq 1$ and raise the question of recurrency to the origin. More precisely, we will give some insight to Georges Polya's Theorem (1921) which, surprisingly, states that the random walk is recurrent only in dimensions 1 and 2. The core of my talk consists in one way of proving this theorem using the (well-known) Borel Cantelli Lemma as well as some Fourier analysis.

Robert BAUMGARTH «*New Integral Criterion for Existence & Completeness of Wave Operators by Stochastic Analysis*»

Abstract. We show a recently proved, new integral criterion for the existence and completeness of the wave operators $W_{\pm}(-\Delta_h, -\Delta_g, I_{g,h})$ corresponding to the Laplace-Beltrami operators $-\Delta_j$, $j = g, h$, that are induced by two quasi-isometric complete Riemannian metrics g and h on an open manifold M . In particular, this result provides a criterion for the absolutely continuous spectra of $-\Delta_g$ and $-\Delta_h$ to coincide. The proof relies on estimates that are obtained using a probabilistic Bismut type formula for the gradient of a heat semigroup. Unlike all previous results, our integral criterion only requires some lower control on the Ricci curvatures and some upper control on the heat kernels, but no control at all on the injectivity radii. All notions will be briefly introduced during the talk as needed concerning the broad audience.