Symmetries in Differential Geometry and Mathematical Physics

Titles and Abstracts

Dmitri Alekseevsky (Brno)

Invariant Kaehler and Kaehler-Einstein metrics on compact cohomogeneity one manifolds

The talk is based on a joint work with Andrea Loi and Fabio Zuddas. Starting from a nice description of indicated metrics in terms of some data on a flag manifold given by F.Podesta and A.Spiro, we obtain a classification of such metrics associated with flag manifolds $F = G/K$ with one or two dimensional center of the stabilizer K.

Helga Baum (HU Berlin)

Global Lorentzian manifolds with special holonomy

Florin Belgun (Hamburg)

Projective curves in conformal geometry

In conformal geometry, small dimensional submanifolds inherit a richer structure than the conformal structure alone. While a conformal structure on a curve is empty, a curve in a conformal manifold has a canonical projective structure. This has been known for over 20 years, but there are almost no examples where the projective parametrizations can be actually computed. We show that every element in the moduli space of closed projective curves can be realized as a conformal geodesic, and, for some classes (some of them exotic, or non-homogeneous), we give explicit realizations in the euclidian plane. Joint work with Andrei Moroianu (Ecole Polytechnique, Palaiseau).

Mohamed Belkhelfa (Mascara)

Symmetry properties of S space form

In this talk we study the pseudo-symmetry and Ricci-pseudo symmetry of S-space-form, $M^{2n+S}(c)$ which reduce, for $S = 1$, to Sasakian space forms which are pseudo symmetric with Deszcz's function 1. We also investigate the pseudo parallel hypersurface of generalized Sasakian space form.

Said Benayadi (Metz)

Left invariant para-Kaehler structures on Lie groups

We study left invariant para-Kaehler structures on Lie groups. We give some interesting examples of this Lie groups and some constructions based on S-matrices. In particular, we study abelian left invariant para-Kaehler structures on Lie groups. In this case the curvatures of the para-Kaehler metric are computed and sufficient conditions to ensure flatness or Ricci-flatness are given. The Lie algebra for which the para-Kaehler metric is Einstein and non Ricci-flat are completly characterised.

Jürgen Berndt (King's College London)

Hyperpolar actions on noncompact symmetric spaces

An isometric action of a connected Lie group H on a Riemannian manifold M is called hyperpolar if there exists a connected closed flat submanifold Σ of M such that Σ meets each orbit of the action and intersects it orthogonally. An elementary example of a hyperpolar action comes from the standard representation of SO_n on $\mathbb{R}^n$. Further examples of hyperpolar actions can be constructed from Riemannian symmetric spaces. Let $M = G/K$ be a Riemannian symmetric space and denote by o a fixed point of the K-action on M. Then the isotropy representation $\pi : K \to O(T_oM)$ of K on the tangent space T_oM of M at o induces a hyperpolar action. Dadok established in 1985 a remarkable, and mysterious, relation between hyperpolar actions on Euclidean spaces and Riemannian symmetric spaces. He proved that for every hyperpolar action on $\mathbb{R}^n$ there exists a Riemannian symmetric space $M = G/K$ with $dimM = n$ such that the orbits of the action on $\mathbb{R}^n$ and the orbits of the K-action on T_oM are the same via a suitable isomorphism $\mathbb{R}^n \to T_oM$. For symmetric spaces of compact type the hyperpolar actions are reasonably well understood. In the talk I want to focus on symmetric spaces of noncompact type where the situation is much more involved because of the noncompactness of the isometry groups. With my collaborators Díaz-Ramos and Tamaru I developed an approach based on Langlands and Chevalley decompositions of parabolic subalgebras of noncompact semisimple Lie algebras. Geometrically this involves horospherical decompositions of symmetric spaces and boundary components of their maximal Satake compactifications. This approach leads to many new examples, partial classifications, and interesting open problems. The talk will be based on these two papers:

1. J. Berndt, J.C. Díaz-Ramos, H. Tamaru: Hyperpolar homogeneous foliations on symmetric spaces of noncompact type, *J. Differential Geom.* **86** (2010), 191-235.

2. J. Berndt, H. Tamaru: Cohomogeneity one actions on symmetric spaces of noncompact type, to appear in *J. Reine Angew. Math.*, published online: DOI: 10.1515/crelle-2012-0002, March 2012.

Vicente Cortés (Hamburg)

Conification of Kähler and hyper-Kähler manifolds

Given a Kähler manifold M endowed with a Hamiltonian Killing vector field Z, we construct a conical Kähler manifold $\hat{M}$ such that M is recovered as a Kähler quotient of $\hat{M}$. Similarly, given a hyper-Kähler manifold (M, g, J_1, J_2, J_3) endowed with a Killing vector field Z, Hamiltonian with respect to the Kähler form of J_1 and satisfying $\mathcal{L}_Z J_2 = -2J_3$, we construct a hyper-Kähler cone $\hat{M}$ such that M is a certain hyper-Kähler quotient of $\hat{M}$. In this way, we recover a theorem by Haydys. Our work is motivated by the problem of relating the supergravity c-map to the rigid c-map. We show that any hyper-Kähler manifold in the image of the c-map admits a Killing vector field with the above properties. Therefore, it gives rise to a hyper-Kähler cone, which in turn defines a quaternionic Kähler manifold. Our results for the signature of the metric and the sign of the scalar curvature are consistent with what we know about the supergravity c-map. This talk is based on joint work with Dmitri Alekseevsky and Thomas Mohaupt, see arXiv:1205.2964.

Liana David (IMAR Bucarest)

Invariant generalized complex structures on Lie groups

In this talk, I will present my joint work with D.V. Alekeevsky on invariant generalized complex structures on Lie groups. After a brief review of basic notions from generalized complex geometry, I will define and study a class (called regular) of invariant generalized complex structures on a real semisimple Lie group G. I will present a classification of regular generalized complex structures, when G is of inner type. If time allows, I will give examples of regular generalized complex structures on semisimple Lie groups of outer type.

Chandrashekar Devchand (Potsdam)

Oxidation of self-duality through heat flow and remixing

Abstract: Using (partial) curvature flows and the transitive action of subgroups of $O(d, \mathbb{Z})$ on the indices $\{1, \ldots, d\}$ of the components of the Yang-Mills curvature in an orthonormal basis, we obtain a nested system of equations in successively higher dimensions d, each implying the Yang-Mills equations on d-dimensional Riemannian manifolds possessing special geometric structures. This 'matryoshka' of self-duality equations contains the familiar self-duality equations on Riemannian 4-folds as well as their generalisations on complex Kähler 3-folds and on 7- and 8-dimensional manifolds with G_2 and $Spin(7)$ holonomy. The matryoshka allows enlargement ('oxidation') to a remarkable system in 12 dimensions invariant under $Sp(3)$.

Marco Freibert (Hamburg)

Cocalibrated structures and the Hitchin flow on Lie algebras

Cocalibrated G_2-structures are the initial values for Hitchin's flow equations whose solutions define Riemannian manifolds with holonomy contained in $Spin(7)$. In this talk I will present classification results for certain classes of Lie algebras which admit such structures. Moreover, I will look at the Hitchin flow on almost Abelian Lie algebras and show that in this case the solutions define Riemannian metrics with holonomy group further reduced to a subgroup of $SU(4)$.

Anton Galaev (Brno)

Decomposition of the covariant derivative of the curvature tensor for pseudo-Riemannian manifolds

The decomposition of the spaces of possible values of the covariant derivative of the curvature tensor for pseudo-Riemannian manifolds with each possible irreducible holonomy group will be discussed. As an application, the result obtained together with D.V. Alekseevsky about a classification of the Lorentzian manifolds with zero second covariant derivative of the curvature tensor will be presented.

Noel Hustler (Edinburgh)

The homogeneity theorem for supergravity backgrounds

We prove the strong homogeneity conjecture for eleven- and ten-dimensional Poincaré supergravity backgrounds. This states that any such background admitting more than half of the maximum number of supergravity Killing spinors must be locally homogeneous and this homogeneity is due precisely to the isometries generated by these Killing spinors.

Stefan Ivanov (Sofia)

The sharp lower bound of the first eigenvalue of the sub-Laplacian on quaternionic contact manifold

We show a Bochner type formula for the sub-laplacian on a compact quaternionic contact manifold. With the help of this formula we establish a version of Lichnerowicz' theorem giving a lower bound of the eigenvalues of the sub-Laplacian under a lower bound on the $Sp(n)Sp(1)$-components of the qc-Ricci curvature. It is shown that in the case of a 3-Sasakian manifold the lower bound is reached if and only if the quaternionic contact manifold is a round 3-Sasakian sphere.

Victor Kac (MIT)

Cyclic elements in semisimple Lie algebras

Ines Kath (Greifswald)

Indefinite symmetric spaces with $\mathrm{G}_{2(2)}$-structure

The compact Lie group $\mathrm{G}_2 \subset \mathrm{SO}(7)$ lies on the list of holonomy groups of irreducible Riemannian manifolds. Each Riemannian manifold whose holonomy group is contained in G_2 is Ricci-flat. In particular, if the holonomy group of a Riemannian symmetric space M is contained in G_2, then M must be flat. In contrast, we will see that there exist indefinite symmetric spaces of signature $(4,3)$ whose holonomy is contained in the split real form $\mathrm{G}_{2(2)} \subset \mathrm{SO}(4,3)$. Moreover, using the method of quadratic extension we determine all indecomposable pseudo-Riemannian symmetric spaces of signature $(4,3)$ with holonomy in $\mathrm{G}_{2(2)}$.

Stefano Marchiafava (Roma "Sapienza")

Some results on CR quaternionic manifolds and related structures

We present some recent results of a collaboration with Stere Ianus, Liviu Ornea and Radu Pantilie, concerning quaternionic geometry. In a general and non metrical framework we introduced the class of CR quaternionic manifolds, which contains the quaternionic manifolds and in dimension three particularizes to, essentially, give the conformal manifolds. We showed that this manifolds have a rich natural Twistor Theory. Moreover we obtained a heaven space construction for quaternionic manifolds and proved that under rather general hypothesis a CR quaternionic manifold can be obtained as a submanifold of a quaternionic manifold; some application to submanifolds of a quaternionic manifolds followed. We developed also a dual point of view by considering the co-CR quaternionic manifolds.

Andrea Spiro (Camerino)

Non-integrable distributions, Cartan connections and equivalence problems

After a review of the classical approaches to the solution of the equivalence problems for G-structures and G-structures adapted to non-integrable distributions, we present a recent result of a joint work with C. Medori on the equivalence problem for a special class of Levi degenerate CR manifolds.

Andrew Swann (Aarhus)

Symmetries of hyperKähler manifolds

I will survey results that classify various symmetry notions for hyperKähler manifolds and emphasise the role of coadjoint orbits. HyperKähler reduction will be discussed and the notion of hyperKähler implosion will be introduced as a mechanism for Abelianising the quotient construction. I will then describe the construction of a universal hyperKähler implosion for $SU(n)$ actions.

Gudlaugur Thorbergsson (Cologne)

Polar actions and foliations

In the talk I will explain recent results on polar actions on compact symmetric spaces and positively curved Riemannian manifolds. I will also talk about polar foliations as a generalization of isoparametric submanifolds. The results are due to several authors.

Luigi Vezzoni (Turin)

Almost Complex structures and the Calabi-Yau equation

The talk is about almost complex structures dominated by symplectic forms. An almost complex structure J is *dominated* by a symplectic form Ω if $G = \Omega(J\cdot, \cdot)$ is a positive tensor. If further G is a Riemannian metric, J is said to be *calibrated* by Ω. There are known many examples of almost complex structures dominated by a symplectic form but not calibrated. Anyway in some special cases it turns out that a J is calibrated if and only if it is dominated. This happens for instance when J is a complex structure on a 4-dimensional compact manifold. A symplectic form together with a dominated J is called a *Hermitian-Symplectic* structure. Such structures were introduced by J. Streets and G. Tian as static solutions to the pluriclosed flow. It is still an open problem to find out an example of a Hermitian-Symplectic structure on a non-Kähler space. In the talk it will be shown that complex nilmanifolds never admit Hermitian-symplectic structures and in that case solutions to the pluriclosed flow are defined for every $t \geqslant 0$.

In a recent paper Donaldson conjectured that in a compact 4-dimensional manifold having $b^+ = 1$ every dominated almost complex structure is also calibrated. He suggested to face this problem using the Calabi-Yau equation. Some partial results about this problem on T^2-fibrations will be shown in the talk.