$\mathcal{W}$-entropy formula, Perelman's Ricci flow and optimal transport on manifolds with weighted measure

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Abstract. In this talk, I will present some recent results in the study of the W-entropy formula, Perelman's Ricci flow and the optimal transport problems on manifolds with weighted measure. After a brief review of Perelman's W-entropy formula for Ricci flow, I will present our result on the $\mathcal{W}$-entropy formula for the heat equation of the weighted Laplacian on manifolds with weighted measure. Then I will present some results on the optimal transport problems for the Fokker-Planck diffusions on manifolds equipped with Perelman's Ricci flow, which can be viewed a natural correspondence of some previous results due to Otto-Villani, von Renesse-Sturm, McCann-Topping and Lott, etc. We point out that there is an interesting similarity between our $\mathcal{W}$-entropy formula for the weighted Laplacian and Lott-Villani and Sturm's result on the monotonicity of the Boltzmann entropy along geodesic on the Wasserstein space over compact Riemannian manifolds. Finally we prove the entropy monotonicity theorem on a family of flows which interpolate the geodesic flow and the gradient flow on the Wasserstein space over compact Riemannian manifolds. This is a joint work with Songzi Li.