RMATH | MATHEMATICS RESEARCH UNIT

UNIVERSITÉ DU LUXEMBOURG

Agence Nationale de la Recherche ANR

Conference ProbaGeo 2013

October 28-31, 2013, University of Luxembourg

Monday, Oct 28 (Campus Limpertsberg, Salle BS 015)

14:30–15:15	*Xiangdong Li*
15:15–16:00	*David Elworthy*
16:00–16:30	Coffee break
16:30–17:15	*Roland Friedrich*
17:15–18:00	*Ionel Popescu*
18:30	Reception

Tuesday, Oct 29 (Campus Kirchberg, Salle Feidert)

09:30–10:15	*Hatem Hajri*	14:30–15:15	*Erwan Hillion*
10:15–11:00	*Xue-Mei Li*	15:15–16:00	*Thierry Lévy*
11:00–11:30	Coffee break	16:00–16:30	Coffee break
11:30–12:15	*Jürgen Angst*	16:30–17:15	*Ivan Nourdin*
12:15–13:00	*Camille Tardif*	17:15–18:00	*Louis H.Y. Chen*
		20:00	Conference dinner

Wednesday, Oct 30 (Campus Limpertsberg, Salle Tavenas)

09:30–10:15	*Karl-Theodor Sturm*	14:30–15:15	*Fabrice Baudoin*
10:15–11:00	*Kazuhiro Kuwae*	15:15–16:00	*Michel Bonnefont*
11:00–11:30	Coffee break	16:00–16:30	Coffee break
11:30–12:15	*Christian Ketterer*	16:30–17:15	*Dejun Luo*
12:15–13:00	*Batu Güneysu*	17:15–18:00	*Koléhè Coulibaly-Pasquier*

Thursday, Oct 31 (Campus Limpertsberg, Salle BS 003)

09:30–10:15	*Janna Lierl*
10:15–10:45	Coffee break
10:45–11:30	*Guillaume Cébron*
11:30–12:15	*Kazumasa Kuwada*

Titles and Abstracts

- **Jürgen Angst** (Université de Rennes 1, France)
'Poisson boundary: a dévissage method'
Abstract. We present a method that allows, under suitable equivariance and regularity conditions, to determine the Poisson boundary of a diffusion starting from the Poisson boundary of a sub-diffusion of the original one.

- **Fabrice Baudoin** (Purdue University, West Lafayette, USA)
'Bakry-Emery meet Villani'
Abstract. We study gradient bounds for solutions of degenerate Fokker-Planck equations on manifolds, including as a special case, the celebrated kinetic Fokker-Planck equation. Our method generalizes to hypoelliptic operators the Bakry-Emery's approach and allows to recover and strengthen hypocoercive estimates obtained by Villani.

- **Michel Bonnefont** (Université Bordeaux 1, France)
'Intertwinning relations for one dimensionnal diffusion semigroups'
Abstract. In this talk, I will present new intertwining relations for diffusion semigroups mainly in the 1-dimensional case. Various applications of these results are investigated, among them the famous variational formula of the spectral gap derived by Chen and Wang together with a new criterion ensuring that the logarithmic Sobolev inequality holds. This is joint work with Aldéric Joulin.

- **Guillaume Cébron** (Université Pierre et Marie Curie, Paris, France)
'Brownian motion and Segal-Bargmann-Hall transform on the linear group in large dimension'
Abstract. The talk will address two topics related to the work of Philippe Biane from the late 1990s: the convergence of the noncommutative distribution of the Brownian motion on the linear group to its free version, and the convergence of the Segal-Bargmann-Hall transform on the unitary group to its free version, as the dimension tends to infinity. The techniques rely on the identification of the Laplace operators at the limit.

- **Louis H. Y. Chen** (National University of Singapore)
'On the error bound in a combinatorial Central Limit Theorem'
Abstract. Let $\{X_{ij}: i,j=1,\ldots,n\}$ be an $n\times n$ array of independent random variables with finite third moments and let π be a random permutation of $\{1,\ldots,n\}$ independent of the X_{ij}. Let $U=\sum_{i=1}^{n} X_{i\pi(i)}$ and $W=(U-\mathbb{E}U)/(\mathrm{Var}(U))^{1/2}$. A third-moment error bound on the Kolmogorov distance with an explicit constant is obtained for the central limit theorem for W by using Stein's method of exchangeable pairs and a concentration inequality. This result is more general than that of Bolthausen (1984), which is on an $n\times n$ array of real numbers and does not have an explicit constant in the error bound. This result also yields a result for sampling without replacement from a finite set of random variables whose means are not necessarily zero. This is more general than the case considered by Wolff (2012), who assumed zero means and obtained a bound on the Wasserstein distance. It is also more general than the case of sampling without replacement from a finite set of real

numbers, considered by Goldstein (2007), who also obtained a bound on the Wasserstein distance. This talk is based on a joint paper with Xiao Fang.

- **Koléhè Coulibaly-Pasquier** (Université de Lorraine, Nancy, France)
'Inhomogeneous diffusion and heat kernel estimates'
Abstract. TBA

- **David Elworthy** (University of Warwick, Coventry, UK)
'A Sard's theorem for stochastic flow measures, and McKean-Singer formulae for certain foliations'
Abstract. Diffusion measures on path spaces of Lie groups and diffeomorphism groups can have nice properties giving rise to a comparatively simple proof that they assign measure zero to the sets of critical values of suitable smooth Fredholm maps. This is relevant to Kusuoka's approach to the McKean-Singer formula for the Euler characteristic of a compact Riemannian manifold in terms of the supertrace of its heat kernel. I will describe this version of Sard's theorem and relate it to a proposed extension of Kusuoka's method to a tangential version of the McKean-Singer formula for a class of foliated manifolds.

- **Roland Friedrich** (Bonn, Germany)
'Geometric Aspects of Free Probability Theory'
Abstract. In this talk we will develop the algebraic and geometric structures underlying Free Probability Theory. This allows us to give the solution to two longstanding major problems in Free Probability, in particular it permits to generalise Voiculescu's S-transform to arbitrary dimensions. This talk is based on recent and ongoing research.

- **Batu Güneysu** (Humboldt-Universität zu Berlin, Germany)
'On the total variation on manifolds with Ricci curvature unbounded from below'
Abstract. In this talk, I will first explain some new global facts about functions with a bounded total variation, that are valid on arbitrary Riemannian manifolds. After that, I will discuss the heat kernel characterization of the total variation on a large class of noncompact Riemannian manifolds with Ricci curvature unbounded from below. This is joint work with Diego Pallara.

- **Hatem Hajri** (Université du Luxembourg)
'Stochastic flows on graphs'
Abstract. Starting from Tanaka's equation on the real line, we define and study some equations on general metric graphs driven by one or many Brownian motions and describe their associated stochastic flows.

- **Erwan Hillion** (Université du Luxembourg)
'Benamou-Brenier curves on graphs'
Abstract. Given a couple f_0, f_1 of finitely supported probability measures on a graph, we construct an interpolating curve $(f_t)_{t\in[0,1]}$ which plays the same role as a W_2 Wasserstein geodesic in continuous spaces. We then study the interplay between the geometry of the graph and the convexity properties of the entropy functional along these curves.

- **Christian Ketterer** (Universität Bonn, Germany)
'Cones over metric measure spaces and the maximal diameter theorem'
Abstract. We present the following result. The (K, N)-cone over some metric measure space satisfies the reduced Riemannian curvature-dimension condition $\mathrm{RCD}^*(KN, N+1)$ if and only if the underlying space satisfies $\mathrm{RCD}^*(N-1, N)$. The proof uses a characterization of reduced Riemannian curvature-dimension bounds by Bochners inequality that was established for metric measure spaces by Erbar, Kuwada and Sturm and independently by Ambrosio, Mondino and Savaré. As corollary of our result and the Cheeger-Gigli-Gromoll splitting theorem we obtain a maximal diameter theorem for metric measure spaces that satisfy a reduced Riemannian curvature-dimension condition.

- **Kazumasa Kuwada** (Ochanomizu University, Tokyo, Japan)
'The entropic curvature-dimension condition and Bochner's inequality'
Abstract. We show the equivalence of the curvature-dimension bounds of Lott-Sturm-Villani (via entropy and optimal transport) and of Bakry-Émery (via energy and Γ_2-calculus) in complete generality for infinitesimally Hilbertian metric measure spaces. In particular, we establish the full Bochner inequality on such metric measure spaces. Moreover, we deduce new contraction bounds for the heat flow on Riemannian manifolds and on mms in terms of the L^2-Wasserstein distance.

- **Kazuhiro Kuwae** (Kumamoto University, Japan)
'On spectral bounds for symmetric Markov chains with coarse Ricci curvatures'
Abstract. We prove an upper estimate of spectral radius for (non-linear) transition operator P over L^p-maps in the framework of symmetric Markov chains on a Polish space with positive lower bound of n-step coarse Ricci curvatures. The target space is a complete separable 2-uniformly convex space with some geometric conditions including the case of CAT(0)-spaces. As consequences, strong L^p-Liouville property for P-harmonic maps, a global Poincaré inequality (spectral gaps) for energy functional over L^2-maps (or functions), and spectral bounds of L^2-generator of Markov chains are presented. This is a joint work with Eiki Kokubo, who was my previous student in master course of Kumamoto University.

- **Thierry Lévy** (Université Pierre et Marie Curie, Paris, France)
'Parking functions and factorisations of permutations'
Abstract. In the course of the study of the limiting distribution of the eigenvalues of a unitary Brownian motion, one is led to enumerating certain paths in the symmetric group. I will present a bijective enumeration of the paths of interest, which involves analogues of parking functions.

- **Xiangdong Li** (Chinese Academy of Sciences, Beijing, China)
'$\mathcal{W}$-entropy formula, Perelman's Ricci flow and optimal transport on manifolds with weighted measure'
Abstract. In this talk, I will present some recent results in the study of the W-entropy formula, Perelman's Ricci flow and the optimal transport problems on manifolds with weighted measure. After a brief review of Perelman's W-entropy formula for Ricci flow,

I will present our result on the $\mathcal{W}$-entropy formula for the heat equation of the weighted Laplacian on manifolds with weighted measure. Then I will present some results on the optimal transport problems for the Fokker-Planck diffusions on manifolds equipped with Perelman's Ricci flow, which can be viewed a natural correspondence of some previous results due to Otto-Villani, von Renesse-Sturm, McCann-Topping and Lott, etc. We point out that there is an interesting similarity between our $\mathcal{W}$-entropy formula for the weighted Laplacian and Lott-Villani and Sturm's result on the monotonicity of the Boltzmann entropy along geodesic on the Wasserstein space over compact Riemannian manifolds. Finally we prove the entropy monotonicity theorem on a family of flows which interpolate the geodesic flow and the gradient flow on the Wasserstein space over compact Riemannian manifolds. This is a joint work with Songzi Li.

- **Xue-Mei Li** (University of Warwick, Coventry, England) *'Smooth dependence of the solution of the backward Kolmogorov equation on the potential'*
Abstract. We study the backward Kolomogorov equation with a zero order term b which is not assumed to be locally bounded. We prove that the solution varies 'smoothly' with respect to the shift of the potential by a continuous path. This allows us to prove a regularizing property of integrating a Brownian functional in time.

- **Janna Lierl** (Universität Bonn, Germany)
'The boundary Harnack principle in fractal spaces'
Abstract. The boundary Harnack principle (BHP) states that the ratio of any two functions, which are positive and harmonic on a domain, is bounded near some part of the boundary where both functions vanish. A given domain may or may not have this property, depending on the geometry of its boundary and the underlying metric measure space. In this talk, I will consider a geometric (scale-invariant) BHP which has interesting applications such as two-sided bounds for the Dirichlet heat kernel. The geometric BHP was proved to hold on inner uniform domains in Euclidean space (Aikawa, Ancona) and in non-fractal Dirichlet spaces of Harnack-type (Gyrya, Saloff-Coste). Extending those works, I will present a scale-invariant BHP for inner uniform domains in metric measure Dirichlet spaces that satisfy volume doubling and weak sub-Gaussian heat kernel bounds. This result applies to fractal-type spaces, e.g. Sierpinski gasket.

- **Dejun Luo** (Chinese Academy of Sciences, Beijing, China)
'A probabilistic proof of the fundamental gap conjecture via the coupling by reflection'
Abstract. Let $\Omega \subset \mathbb{R}^n$ be a strictly convex domain with smooth boundary and diameter D. The fundamental gap conjecture claims that if $V : \bar{\Omega} \to \mathbb{R}$ is convex, then the spectral gap of the Schrödinger operator $-\Delta + V$ with Dirichlet boundary condition is no less than $3\pi^2/D^2$. Recently this conjecture was proved by Andrews and Clutterbuck in [J. Amer. Math. Soc. 24 (2011), no. 3, 899–916] using analytic methods. In this talk we shall give a probabilistic proof via the coupling by reflection of the diffusion processes. This is a joint work with Fuzhou Gong and Huaiqian Li.

- **Ivan Nourdin** (Université de Lorraine, Nancy, France)
'The Optimal Fourth Moment Theorem'
Abstract. I will explain how to compute the exact rates of convergence in total variation associated with the celebrated Fourth Moment Theorem by Nualart and Peccati, stating that a sequence of random variables living in a fixed Wiener chaos verifies a central limit theorem if and only if the sequence of the corresponding fourth cumulants converges to zero. If time permits, I will also provide an explicit illustration based on the Breuer-Major CLT for Gaussian-subordinated random sequences. This will be based on several joint works with Giovanni Peccati.

- **Ionel Popescu** (Georgia Institute of Technology, Atlanta, USA)
'Shy and Fixed distance couplings on Riemanian manifolds'
Abstract. We show that on any Riemannian manifold with the Ricci curvature non-negative (and some other technical conditions) we can construct a coupling of two Brownian motions which are staying fixed distance for all times. We show a more general version of this for the case of Ricci bounded from below uniformly by a constant k. In the terminology of Burdzy, Kendall and others, a shy coupling is a coupling in which the Brownian motions do not couple in finite time with positive probability. What we construct here is a strong version of shy couplings on Riemannian manifolds. On the other hand, this can be put in contrast with some results of von Renesse and K. T. Sturm which give a characterization of the lower bound on the Ricci curvature in terms of couplings of Brownian motions and our construction optimizes this choice in a way which will be explained. This is joint work with Mihai N. Pascu.

- **Karl-Theodor Sturm** (Universität Bonn, Germany)
'Geometric Analysis on the Space of Metric Measure Spaces'
Abstract. The space of all metric measure spaces (X, d, m) plays an important role in image analysis, in the investigation of limits of Riemannnian manifolds and metric graphs as well as in the study of geometric flows that develop singularities. We show that this "space of spaces" – equipped with the L^2-distortion distance – is a challenging object of geometric interest in its own. In particular, we show that it has nonnegative curvature in the sense of Alexandrov. Geodesics and tangent spaces are characterized in detail. Moreover, classes of semiconvex functionals and their gradient flows on the space of spaces are presented.

- **Camille Tardif** (Université Pierre et Marie Curie, Paris, France)
'Identifying Poisson boundaries: applications of the dévissage method'
Abstract. We apply the dévissage method to compute the Poisson boundary of several examples of diffusion processes: Brownian motion on rotationally symmetric manifolds, relativistic diffusion in Minkowski space-time and spatially flat Robertson-Walker space-times.