Uniform volume doubling for left-invariant Riemannian geometries on SU(2)

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Abstract. We show that there is a uniform volume doubling constant for all left-invariant Riemannian geometries on the compact Lie group SU(2). This implies uniformity in a wide variety of spectral properties and functional inequalities for this family of geometries, including Poincaré and parabolic Harnack inequalities. Classical results based on curvature are not available in this setting, since there is no uniform lower bound on the Ricci curvatures of these metrics, even after rescaling to constant diameter. A key idea in the proof is to study the size and shape of metric balls by a careful analysis of the Campbell–Baker–Hausdorff–Dynkin–Strichartz formula. We conjecture that the same uniform doubling result holds for all other compact connected Lie groups.
This is joint work with Maria Gordina (University of Connecticut) and Laurent Saloff-Coste (Cornell).