W-entropy formulas on super Ricci flows and Langevin deformation on Wasserstein space over Riemannian manifolds

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Abstract. In this talk, we give an overview of our recent works on the study of the W-entropy for the heat equation of the Witten Laplacian on super-Ricci flows and the Langevin deformation on the Wasserstein space over Riemannian manifolds. Inspired by Perelmans seminal work on the Ricci flow, we proved the W-entropy formula for the heat equation of the Witten Laplacian on complete Riemannian manifolds with the $CD(K, m)$-condition and for the heat equation of the time dependent Witten Laplacian on compact manifolds equipped with a (K, m)-super Ricci flow, where $m \in [n, \infty]$ and $K \in \mathbb{R}$. Furthermore, we proved an analogue of the W-entropy formula for the Wasserstein geodesic flow which corresponds to the optimal transportation problem on Riemannian manifolds, which recaptures a previous result due to Lott and Villani on the displacement convexity of the Boltzmann-Shannon type entropy on Riemannian manifolds with non-negative Ricci curvature. We introduce the Langevin deformation of geometric flows, which interpolate the geodesic flow and the gradient flows on the Wasserstein space over Riemannian manifolds, and prove the W-entropy formula for the Langevin deformation. Finally, we make a discussion on the W-entropy for the Ricci flow from the point of view of statistical mechanics and probability theory. This is a joint work with Songzi Li.