Fonds National de la Recherche Luxembourg

UNIVERSITÉ DU LUXEMBOURG

RMATH | MATHEMATICS RESEARCH UNIT

Conference GeoProb 2017

July 10–14, 2017, University of Luxembourg

Monday, July 10 (Campus Belval, MSA 3.530)

09:30–10:15	*Fabrice Baudoin*	14:30–15:15	*Feng-Yu Wang*
10:15–10:45	Coffee break	15:15–15:45	Coffee break
10:45–11:30	*Thierry Lévy*	15:45–16:30	*Xiaodong Cao*
11:30–12:15	*Yves Le Jan*	16:30–17:15	*Lei Ni*
		17:15–18:00	*Xue-Mei Li*

Tuesday, July 11 (Campus Belval, MSA 3.530)

09:30–10:15	*David Elworthy*	14:30–15:15	*Masha Gordina*
10:15–10:45	Coffee break	15:15–15:45	Coffee break
10:45–11:30	*Robert Neel*	15:45–16:30	*Nathaniel Eldredge*
11:30–12:15	*Ugo Boscain*	16:30–17:15	*Nizar Demni*
		17:15–18:00	*Ana Bela Cruzeiro*

Wednesday, July 12 (Campus Belval, MSA 3.530)

09:30–10:15	*Kazuhiro Kuwae*		
10:15–10:45	Coffee break		
10:45–11:30	*Kohei Suzuki*	16:45–18:45	*Guided City Tour*
11:30–12:15	*Bang-Xian Han*	19:00–22:00	*Conference Dinner*

Thursday, July 13 (Campus Belval, MSA 3.530)

09:30–10:15	*Elton Hsu*	14:30–15:15	*Xiangdong Li*
10:15–10:45	Coffee break	15:15–15:45	Coffee break
10:45–11:30	*Nicolas Juillet*	15:45–16:30	*Roland Friedrich*
11:30–12:15	*Max Fathi*	16:30–17:15	*Xin Chen*
		17:15–18:00	*Hongxin Guo*

Friday, July 14 (Campus Belval, MSA 3.530)

09:30–10:15	*Marc Arnaudon*
10:15–11:00	*Yi Li*
11:00–11:30	Coffee break
11:30–12:15	*Bo Wu*
12:15–13:00	*Ionel Popescu*

Titles and Abstracts

- **Marc ARNAUDON** (Université Bordeaux 1, France, France)

  "An entropic interpolation problem for incompressible viscid fluids"

  Abstract. In view of studying incompressible inviscid fluids, Brenier introduced in the late 80's a relaxation of a geodesic problem addressed by Arnold in 1966. Instead of inviscid fluids, the talk will be devoted to incompressible viscid fluids. A natural analogue of Brenier's problem is introduced, where generalized flows are no more supported by absolutely continuous paths, but by Brownian sample paths. It turns out that this new variational problem is an entropy minimization problem with marginal constraints entering the class of convex minimization problems. The talk will explore the connection between this variational problem and Brenier's original problem. Its dual problem is derived and the general shape of its solution is described. Under the restrictive assumption that the pressure is a nice function, the kinematics of its solution is made explicit and its connection with the Navier-Stokes equation is established.

- **Fabrice BAUDOIN** (University of Connecticut, USA)

  "Sub-Laplacian Comparison theorems on totally geodesic Riemannian Foliations"

  Abstract. We develop a variational theory of geodesics for the canonical variation of the metric of a totally geodesic foliation. As a consequence, we obtain comparison theorems for the horizontal and vertical Laplacians. In the case of Sasakian foliations, or more generally H-type foliations, we show that a sharp sub-Laplacian comparison theorem for the sub-Riemannian distance may be obtained as a limit of sub-Laplacian comparison theorems for the Riemannian distances approximations. This is joint work with E. Grong, K. Kuwada and A. Thalmaier.

- **Ugo BOSCAIN** (CMAP, École Polytechnique, France)

  "Diffusion in almost-Riemannian geometry"

  Abstract. In this talk I will review certain analytical and geometrical results in 2D-almost-Riemannian geometry. Two-dimensional almost-Riemannian structures are generalized Riemannian structures on surfaces for which a local orthonormal frame is given by a Lie bracket generating pair of vector fields that can become collinear. On the singular set (i.e. where these vector fields become collinear) all Riemannian quantities explodes, but geodesics are still well defined and smooth. Generically the singular set is a smooth curve. In this talk I will review certain geometric properties (normal forms, Gauss-Bonnet theorem) and I will discuss the problem of the Schrödinger and of the heat equation. In particular I will discuss if the singular set acts of not as a barrier for the heat evolution. Some open problems will be also presented.

- **Xiaodong CAO** (Cornell University, USA)

  "Einstein four-manifolds with pinched sectional curvature"

  Abstract. In this talk, I will start with an introduction to Einstein 4-manifolds. Then I will discuss some earlier results on classification of the positive case. Finally I will mention some recent development in this area.

- **Xin CHEN** (Shanghai Jiao Tong University, China)

  "Functional inequalities on loop space over a general non-compact manifold"

  Abstract. We will prove several asymptotic gradient estimates for heat kernel on a general non-compact manifold. Based on these estimates, we could construct an O-U Dirichlet form on loop space over a general non-compact manifold which is only complete and stochastically complete. Moreover, a local log-Sobolev inequality and a log-Sobolev inequality with potential term will also be established. This talk is based on a joint work with Xue-Mei Li and Bo Wu.

- **Ana Bela CRUZEIRO** (Technical University of Lisbon, Portugal)

  "Camassa-Holm and Leray-alpha equations: a stochastic approach"

  Abstract. We derive the periodic incompressible viscous Camassa-Holm equation and the Leray-alpha equations via stochastic variational principles. We discuss the existence of solution for these equations in the space H^1 using the probabilistic characterization. This is joint work with Guoping Liu.

- **Nizar DEMNI** (Université de Rennes, France)

  "Magnetic Laplacians, quasi-infinitely divisible distributions and polyanalytic determinantal processes"

  Abstract. In this talk, we construct quasi-infinitely distributions from eigenfunctions of Magnetic Laplacians in the Euclidean plane and in the hyperbolic disc with constant magnetic field. Doing so leads to generalized Poisson and negative binomial distributions for which we give the Levy-Khintchine type decompositions. We will also shed the light on the relation between these Laplacians, the Sub-Laplacian on the Heisenberg group, the Maass operator and the geometric Brownian motion. Finally, we introduce the polyanalytic determinantal processes and exhibit their rigidity in the sense of Ghosh and Peres.

- **Nathaniel ELDREDGE** (University of Northern Colorado, USA)

  "Uniform volume doubling for left-invariant Riemannian geometries on SU(2)"

  Abstract. We show that there is a uniform volume doubling constant for all left-invariant Riemannian geometries on the compact Lie group SU(2). This implies uniformity in a wide variety of spectral properties and functional inequalities for this family of geometries, including Poincaré and parabolic Harnack inequalities. Classical results based on curvature are not available in this setting, since there is no uniform lower bound on the Ricci curvatures of these metrics, even after rescaling to constant diameter. A key idea in the proof is to study the size and shape of metric balls by a careful analysis of the Campbell–Baker–Hausdorff–Dynkin–Strichartz formula. We conjecture that the same uniform doubling result holds for all other compact connected Lie groups.

  This is joint work with Maria Gordina (University of Connecticut) and Laurent Saloff-Coste (Cornell).

- **David ELWORTHY** (University of Warwick, Coventry, UK)

  "Sums of squares of vector fields, generalised Weitzenböck formulae, and higher order derivative formula"

  Abstract. There is a routine for obtaining path integral formulae for derivatives of smooth heat semigroups, and for certain heat semigroups acting on differential forms etc, established some time ago by myself, LeJan, & XueMei Li. Following a description of this in its general form, I will discuss its applicability to higher order derivatives by looking at some special examples.

- **Max FATHI** (Institut de Mathématiques de Toulouse, France)

  "Stein kernels, optimal transport and the central limit theorem"

  Abstract. Stein kernels are a way of comparing a given probability measure with the standard Gaussian measure, introduced by Stein as a way of obtaining rates of convergence in the central limit theorem. In this talk, I will discuss some properties of those kernels, an existence result and recent work on a connection with the theory of optimal transport. As a consequence, we obtain various quantitative versions of the central limit theorem. Partly based on joint work with Thomas Courtade and Ashwin Pananjady.

- **Roland FRIEDRICH** (Saarland University, Germany)

"Algebraic and geometric aspects of infinitely divisible probability measures"

Abstract. The space of infinitely divisible probability measures on the real line is a well studied object in both classical and non-commutative probability theory. But, as we will show, it also possesses a surprisingly rich algebraic and geometric structure. This talk is based on recent results.

- **Masha GORDINA** (University of Connecticut, USA)

"Couplings for hypoelliptic diffusions"

Abstract. Coupling is a way of constructing Markov processes with prescribed laws on the same probability space. It is known that the rate of coupling (how fast you can make two processes meet) of elliptic/Riemannian diffusions is connected to the geometry of the underlying space. In this talk we consider coupling of hypoelliptic diffusions (diffusions driven by vector fields satisfying Hörmander's condition). S. Banerjee and W. Kendall constructed successful Markovian couplings for a large class of hypoelliptic diffusions. We construct a non-Markovian coupling of Brownian motions on the Heisenberg group, and then use this coupling to prove analytic gradient estimates for harmonic functions for the sub-Laplacian. This talk is based on the joint work with Sayan Banerjee and Phanuel Mariano.

- **Hongxin GUO** (Wenzhou University, Zhejiang, China)

"Constructing monotone quantities in geometric flows via the Boltzmann entropy"

Abstract. We present a method to construct monotone quantities in various geometric flows by studying the Boltzmann entropy of some probability measure which comes naturally from the positive solution of an adjoint heat equation of the flow.

- **Bang-Xian HAN** (University of Bonn, Germany)

"Angles between curves in metric measure space"

Abstract. We propose a new notion of angle between two curves in the framework of metric (and metric measure) spaces. More precisely, we give a new notion of angle between two curves in a metric space. Such a notion has a natural interplay with optimal transportation and is particularly well suited for metric measure spaces satisfying the curvature-dimension condition. Indeed one of the main results is the validity of the cosine formula on $\mathrm{RCD}(K,N)$ metric measure spaces. As a consequence, the new introduced notions are compatible with the corresponding classical ones for Riemannian manifolds, Ricci limit spaces and Alexandrov spaces. This is a joint work with Andrea Mondino.

- **Elton P. HSU** (Northwestern University, USA)

"Functional Inequalities for a Brownian motion of a time-dependent Riemannian metric"

Abstract. We will discuss functional inequalities (Poincaré inequality, logarithmic Sobolev inequality, Beckner's inequality, etc.) for the law of Brownian motion on a compact smooth manifold with a time-dependent Riemannian metric. This topic is closely related to the Ricci flow theory in differential geometry. Some recent contributions by Aaron Naber on the sharp constants for functional inequalities will also be discussed.

- **Nicolas JUILLET** (Université de Strasbourg, France)

"Examples in relation with a metric Ricci flow"

Abstract. Gigli and Mantegazza have observed how optimal transport and heat diffusion allow to describe the direction of the Ricci flow uniquely from the metric aspects of Riemannian manifolds.

The goal is to reformulate the Ricci flow so that it also makes sense for metric spaces. I will present investigations and results obtained with Matthias Erbar (University of Bonn) that concerns some non-Riemannian limits of Riemannian manifolds, in particular the Euclidean cone and the Heisenberg group.

- **Kazuhiro KUWAE** (Fukuoka University, Japan)

"Radial processes on $\mathsf{RCD}^*(K,N)$*-spaces"*

Abstract. I will talk on a stochastic expression of radial processes of Brownian motions on $\mathsf{RCD}^*(K,N)$-spaces. The expression holds under the law for all starting point provided the reference point satisfies a condition. This condition is satisfied for almost everywhere reference point on $\mathsf{RCD}^*(K,N)$-space. This is a joint work with Kazumasa Kuwada.

- **Yves LE JAN** (Université Paris-Sud, Orsay, France)

"On the topology of Markov loops"

Abstract. After introducing Poissonian ensembles of Markov loops on graphs and Brownian loops on manifolds, we present a few results about their distribution in different topological types.

- **Thierry LÉVY** (Université Pierre et Marie Curie, Paris, France)

"The Makeenko-Migdal equations and the master field"

Abstract. I will report on recent progress in 2-dimensional Yang-Mills theory, due to Driver-Gabriel-Hall-Kemp who greatly simplified my proof of the Makeenko-Migdal equations, and to Dahlqvist-Norris who proved the existence of a large N limit of the holonomy field (the so-called master field) on the sphere.

- **Xiangdong LI** (Chinese Academy of Sciences, Beijing, China)

"W*-entropy formulas on super Ricci flows and Langevin deformation on Wasserstein space over Riemannian manifolds"*

Abstract. In this talk, we give an overview of our recent works on the study of the W-entropy for the heat equation of the Witten Laplacian on super-Ricci flows and the Langevin deformation on the Wasserstein space over Riemannian manifolds. Inspired by Perelmans seminal work on the Ricci flow, we proved the W-entropy formula for the heat equation of the Witten Laplacian on complete Riemannian manifolds with the $\mathrm{CD}(K,m)$-condition and for the heat equation of the time dependent Witten Laplacian on compact manifolds equipped with a (K,m)-super Ricci flow, where $m \in [n,\infty]$ and $K \in \mathbb{R}$. Furthermore, we proved an analogue of the W-entropy formula for the Wasserstein geodesic flow which corresponds to the optimal transportation problem on Riemannian manifolds, which recaptures a previous result due to Lott and Villani on the displacement convexity of the Boltzmann-Shannon type entropy on Riemannian manifolds with non-negative Ricci curvature. We introduce the Langevin deformation of geometric flows, which interpolate the geodesic flow and the gradient flows on the Wasserstein space over Riemannian manifolds, and prove the W-entropy formula for the Langevin deformation. Finally, we make a discussion on the W-entropy for the Ricci flow from the point of view of statistical mechanics and probability theory. This is a joint work with Songzi Li.

- **Xue-Mei LI** (University of Warwick, Coventry, England)

"Stochastic averaging"

Abstract. We discuss stochastic averaging on manifolds and related problems.

- **Yi LI** (University of Luxembourg)

"Heat kernel estimates and local curvature estimates for Ricci-harmonic flow"

Abstract. In this talk I mainly give two estimates for Ricci-harmonic flow. The first one is the recent joint work with Guoqiang Wu on heat kernel, while the second one is the very recent result on local curvature estimates. At the end, I will discuss an analog bounded L^2 curvature problem (related to Ricci-harmonic flow) in the general relativity.

- **Robert NEEL** (Lehigh University, USA)

"Random walks, Laplacians, and volumes in sub-Riemannian geometry"

Abstract. We study a variety of random walks on sub-Riemannian manifolds and their diffusion limits, which give, via their infinitesimal generators, second-order operators on the manifolds. A primary motivation is the lack of a canonical Laplacian in sub-Riemannian geometry, and thus we are particularly interested in the relationship between the limiting operators, the geodesic structure, and operators which can be obtained as divergences with respect to various choices of volume.

- **Lei NI** (University of California, San Diego, USA)

"Flow by the power of the Gauss curvature"

Abstract. We prove that convex hypersurfaces in $\mathbb{R}^{n+1}$ contracting under the flow by any power $\alpha > \frac{1}{n+2}$ of the Gauss curvature converge (after rescaling to fixed volume) to a limit which is a smooth, uniformly convex self-similar contracting solution of the flow (soliton). Under additional central symmetry of the initial body we prove that the limit is the round sphere for $\alpha \geq 1$. Recent work of Brendle-Choi-Daskalapoulos asserts that the soliton is the round sphere for $\alpha > \frac{1}{n+2}$. This is a joint work with Ben Andrews and Pengfei Guan.

- **Ionel POPESCU** (Georgia Institute of Technology, Atlanta, USA)

"Deterministic distance couplings on Riemannian manifolds"

Abstract. I will discuss the construction of couplings of Brownian motions on Riemannian manifolds for which the distance between them is deterministic. In the case of model manifolds of constant curvature we give a complete characterization of such distance functions and couplings.

- **Kohei SUZUKI** (University of Bonn, Germany)

"Convergence of Diffusion Processes on RCD *Spaces"*

Abstract. We talk about the weak convergence of laws of symmetric and non-symmetric diffusion processes on RCD spaces in connection with geometric convergences of the underlying spaces.

- **Feng-Yu WANG** (Beijing Normal University, China)

"Distribution dependent SDE*s for Landau type equations"*

Abstract. The distribution dependent stochastic differential equations (DDSDEs) describe stochastic systems whose evolution is determined by both the microcosmic site and the macrocosmic distribution of the particle. The density function associated with a DDSDE solves a nonlinear PDE. Due to the distribution dependence, some standard techniques developed for SDEs do not apply. By iterating in distributions, a strong solution is constructed using SDEs with control. The distribution of solutions is identified with a nonlinear semigroup for probability measures. The exponential contraction as well as Harnack inequalities and applications are investigated.

- **Bo WU** (Fudan University, Shanghai, China)

"Characterizations of Ricci Curvature and Second Fundamental Form on Riemannian Manifold"

Abstract. In prior work of Aaron Naber, the uniform bounds of Ricci curvature on a Riemannian manifold are characterized by using functional inequalities with respect to Malliavin gradient on path space. The goal of our talk is twofold: on one side we extend these results to a general Riemannian manifold (possible with boundary). On the other we obtain some similar results if Malliavin gradient is replaced by L^2-gradient.