Holomorphic functions and subelliptic heat kernels over Lie groups

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(Based on joint work with Laurent Saloff-Coste and Leonard Gross) A Hermitian form q on the dual space, $\mathfrak{g}^*$, of a Lie algebra, $\mathfrak{g}$, of a Lie group, G, determines a Laplacian, Δ, on G. Assuming Hörmander's condition for hypoellipticity, the subelliptic heat semigroup, $e^{t\Delta/4}$, is given by convolution by a C^∞ probability density ρ_t. Analogous to earlier work in the strongly elliptic case, we are able to show that if G is complex, connected, and simply connected then the Taylor expansion defines a unitary map from the space of holomorphic functions in $L^2\left(G, \rho_t\right)$ onto (a subspace of) the dual of the universal enveloping algebra in the norm induced by q. This work is related to an extension of the bosonic Fock space to the noncommutative Lie group setting.