Maximum Likelihood Characterization Theorems: a generalization of Gauss' principle

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In probability and statistics, a characterization theorem occurs whenever a given law or a given class of laws is the only one which satisfies a certain property. A classical research field, initiated by Gauss and Poincaré, is concerned with Maximum Likelihood characterizations. Indeed, Gauss in 1809 proved the following result, nowadays known as Gauss' principle: the Maximum Likelihood Estimator (MLE) of the location parameter in a location family of probability distributions always (this notion will become clearer in the talk) coincides with the sample arithmetic mean $\frac{1}{n}\sum_{i=1}^{n} X_i$ if and only if the sampled observations $X_1, \ldots, X_n$ are drawn from a normal/Gaussian population. It is through this principle that Gauss discovered the celebrated probability distribution which carries his name. Poincaré in 1912 has similarly characterized members of the vast exponential family. Several extensions of their results have since emerged in the literature. To cite but a few, Teicher (1961) studies an MLE characterization of the normal distribution with respect to the scale (or dispersion) parameter, while Ferguson (1962) characterizes a one-parameter generalized normal distribution via the MLE of its location parameter. Besides these rather "linear" setups, MLE characterizations have also been examined in "non-linear" cases, such as spherical distributions (i.e., distributions taking their values only on the unit hypersphere) in dimensions $k > 1$ (e.g., by von Mises 1918 or Bingham and Mardia 1975). Each such characterization theorem has been obtained by means of ad hoc methods, developed for each case separately.

In this talk, we provide a new and unified perspective on this literature by showing that all these results are different instances of a single phenomenon. By doing so, we will on the one hand provide a better understanding of the similarities and differences between existing characterizations and on the other hand show how our point of view allows to easily construct several new MLE characterizations.