

**Optimal Transportation, Ricci Curvature and Diffusions
on the L2-Wasserstein Space
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We introduce and analyze generalized Ricci curvature bounds for metric measure spaces (M, d, m) , based on convexity properties of the relative entropy $\text{Ent}(\cdot|m)$. For Riemannian manifolds, $\text{Curv}(M, d, m) \geq K$ if and only if $\text{Ric}_M \geq K$ on M ; for the Wiener space, $\text{Curv}(M, d, m) = 1$.

One of the main results is that these lower curvature bounds are stable under (e.g. measured Gromov-Hausdorff) convergence. This solves one of the basic problems in this field, open for many years.

Furthermore, we introduce a (more restrictive) curvature-dimension condition $\text{CD}(K, N)$ which implies sharp versions of the Brunn-Minkowski inequality, of the Bishop-Gromov volume comparison theorem and of the Bonnet-Myers theorem. Moreover, it allows to construct a canonical Dirichlet form with Gaussian bounds for the corresponding heat kernel.

Finally, we indicate how to construct a canonical reversible process on the L^2 -Wasserstein space of probability measures $\mathcal{P}(\mathbb{R})$, regarded as an infinite dimensional Riemannian manifold. This process has an invariant measure \mathbb{P}^β which may be characterized as the 'uniform distribution' on $\mathcal{P}(\mathbb{R})$ with weight function $\frac{1}{Z} \exp(-\beta \cdot \text{Ent}(\cdot|m))$ where m denotes a given finite measure on \mathbb{R} . One of the key results is the quasi-invariance of this measure \mathbb{P}^β under push forwards $\mu \mapsto h_*\mu$ by means of smooth diffeomorphisms h of \mathbb{R} .