International Program on Regularity Structures and Stochastic Systems

July 9th-August 3rd, 2018, Beijing, China

Academy of Mathematics and Systems Science, CAS

Conference manual

Sponsors
AMSS, Chinese Academy of Sciences
National Natural Science Foundation of China
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I. Conference Presentation

Organizer:
Academy of Mathematics and Systems Science, CAS
Imperial College London (London, UK)

Registration: 14:00-20:00 on July 9 @ New Office Building, AMSS, CAS

Date: 2018.07.09—2018.08.03

Date of Workshop: 07.10—07.14

Venue: New Office Building, AMSS, CAS
  Mini Course and Seminars: Room 204 & Room 109
  Workshop: Room 204

Homepage: https://www.wjx.top/jq/22713673.aspx

Hotel for Conference Speakers:
  Junma International Hotel（骏马国际酒店）

Scientific Committee:
Zhi-Ming Ma, AMSS, CAS (Co-Chair)
Martin Hairer, Imperial College London (Co-Chair)
Fuzhou Gong, AMSS, CAS
Xuemei Li, Imperial College London

Local Organizers:
Fuzhou Gong
Yan Fu
Kai He
Liping Li
Yuan Liu
Dejun Luo
Yongsheng Song

Sponsors
Academy of Mathematics and Systems Science, CAS
National Natural Science Foundation of China, NSFC 11688101, "Geometry, analysis, and computation on manifolds"
# II. Conference Schedule

## Calendar of RSSS

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<tr>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thu</th>
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<th>Sat</th>
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<tr>
<td>Jul 9</td>
<td>Registration</td>
<td>14:00-20:00</td>
<td>New Office Building</td>
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<td>Jul 10—Jul 14</td>
<td>WORKSHOP</td>
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**Time Table**

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<tr>
<th>Class</th>
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<tbody>
<tr>
<td>Class 1</td>
<td>09:15-10:15</td>
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<td>Class 2</td>
<td>10:45-11:45</td>
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**LUNCH**

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<th>Class</th>
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<tr>
<td>Class 3</td>
<td>13:30-14:30</td>
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<tr>
<td>Class 4</td>
<td>15:00-16:00</td>
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</tbody>
</table>

**Venue**

**Workshop:** New Office Building, AMSS, CAS  
Room 204

**Mini Courses:** New Office Building, AMSS, CAS  
Room 204: C1  
Room 109: C2, C3, C4, C5, C6, C7

**Mini Courses**

- C1: Martin Hairer
- C2: Yvain Bruned
- C3: Thomas James Holding
- C4: Konstantin Matetski
- C5: Hao Shen
- C6: Weijun Xu
- C7: Xiangchan Zhu

**Notice:**

On July 18, Prof. Fraydoun Rezakhanlou will give a talk titled “Kinetic Limit for Interacting Particle Systems”.

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## Workshop Schedule

**Date:** Jul 10th, Tuesday  
**Venue:** Room 204 of New Office Building, AMSS, CAS

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<tr>
<th>Time</th>
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<th>Chair</th>
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| 8:10-8:30  | Opening Ceremony  
Zhiming Ma (Chairman of the Academic Committee of AMSS, CAS)  
Xuemei Li (Chair in Probability and Stochastic Analysis, IC) | Jia-an Yan                 |
| 8:30-9:15  | David Elworthy (University of Warwick, UK)  
Title: Towards higher order derivative formulae | Zhiming Ma                 |
| 9:15-10:00 | Kening Lu (Brigham Young University)  
Title: TBA |                            |
| 10:00-10:15| **Tea Break**                                                          |                            |
| 10:15-11:00| Tusheng Zhang (University of Manchester, UK)  
Title: Small time asymptotics of Brownian motion with singular drifts | Kening Lu                  |
| 11:00-11:45| Zhenqing Chen (University of Washington)  
Title: A priori Holder estimate for non-local parabolic SPDE |                           |
| 14:00-14:45| Lunch  
Marc Arnaudon (University of Bordeaux)  
Title: A duality formula and a particle Gibbs sampler for continuous time Feynman-Kac measures on path spaces | Federica Dragoni |
| 14:45-15:30| Nicolas Peter Dirr (Cardiff University, UK)  
Title: Gradient flows and stochastic particle systems |                            |
| 15:30-15:45| **Tea Break**                                                          |                            |
| 15:45-16:30| Federica Dragoni (Cardiff University, UK)  
Title: Stochastic Homogenization in Carnot Groups | Nicolas Peter Dirr |
| 16:30-17:15| Hiroshi Kawabi (Keio University, Yokohama, Japan)  
Title: Functional central limit theorems for non-symmetric random walks on nilpotent covering graphs |                            |
| 17:15-18:00| Stefania Ugolini (University of Milan)  
Title: A stochastic approach to Bose-Einstein Condensation |                           |
<p>| 18:30      | <strong>Dinner</strong>                                                              |                            |</p>
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<tr>
<th>Time</th>
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<tr>
<td>8:30-9:15</td>
<td>Michael Röckner (Bielefeld University and AMSS, CAS) Title: Nonlinear Fokker-Planck-Kolmogorov equations and distribution dependent SDE</td>
<td>Nizar Touzi</td>
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<tr>
<td>9:15-10:00</td>
<td>Anton Thalmaier (University of Luxembourg) Title: Characterization of Ricci curvature and Ricci flow by Brownian motion</td>
<td>Michael Röckner</td>
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<tr>
<td>10:00-10:15</td>
<td>Tea Break</td>
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<td>10:15-11:00</td>
<td>Nizar Touzi (Ecole Polytechnique, France) Title: Branching particles representation for nonlinear PDEs</td>
<td>Michael Röckner</td>
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<td>11:00-11:45</td>
<td>Hirofumi Osada (Kyushu University, Japan) Title: Infinite-dimensional stochastic differential equations with symmetry</td>
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<td></td>
<td>Lunch</td>
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<td>14:00-14:45</td>
<td>Thierry Lévy (Sorbonne Université) Title: Quantum spanning forests</td>
<td>Barbara Rüdiger</td>
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<tr>
<td>14:45-15:30</td>
<td>Seiichiro Kusuoka (Okayama University) Title: The invariant measure and flow associated to the Phi4-quantum field model on the three-dimensional torus</td>
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<tr>
<td>15:30-15:45</td>
<td>Tea Break</td>
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<tr>
<td>15:45-16:30</td>
<td>Barbara Rüdiger (University Wuppertal) Title: The Boltzmann Process</td>
<td>Thierry Lévy</td>
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<tr>
<td>16:30-17:15</td>
<td>Sonia Mazzucchi (University of Trento, Italy) Title: Generalized Feynman-Kac formulae</td>
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<td>17:15-18:00</td>
<td>Martin Grothaus (Technical University of Kaiserslautern, Germany) Title: Approximation of the stochastic heat equation with sticky reflected boundary condition</td>
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<tr>
<td>18:30</td>
<td>Dinner</td>
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Date: Jul 12th, Thursday  
Venue: Room 204 of New Office Building, AMSS, CAS

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<tr>
<th>Time</th>
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| 8:30-9:15     | Yuri Kifer (Hebrew University, Jerusalem)  
Title: Geometric law for multiple returns until a hazard | Paulo Régis Caron Ruffino  |
| 9:15-10:00    | Fengyu Wang (Tianjin University)  
Title: Estimates of Invariant Probability Measures for Singular SDEs |                           |
| 10:00-10:15   | **Tea Break**                                                          |                            |
| 10:15-11:00   | Alain-Sol Sznitman (ETH Zurich)  
Title: On macroscopic holes in some dependent percolation models | Fengyu Wang                |
| 11:00-11:45   | Paulo Régis Caron Ruffino (University of Campinas, Brazil)  
Title: Geodesic jumps in non-continuous SDE: applications to an averaging principle on foliated space |                           |
| 14:00-14:45   | James Thompson (University of Luxembourg)  
Title: Quantitative gradient estimates by Bismut formulae | Ben Hambly                |
| 14:45-15:30   | Bin Xie (Shinshu University)  
Title: On the space-time white noise driven SPDE with reflection |                            |
| 15:30-15:45   | **Tea Break**                                                          |                            |
| 15:45-16:30   | Ben Hambly (University of Oxford)  
Title: SPDEs and systemic risk | Bin Xie                   |
| 16:30-17:15   | Atsushi Atsuji (Keio University, Japan)  
Title: Leafwise Brownian motions and some function theoretic properties of laminations |                           |
| 17:15-18:00   | Hao Shen (Columbia University/University of Wisconsin - Madison)  
Title: TBA |                            |
<p>| 18:30         | <strong>Dinner</strong>                                                             |                            |</p>
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<tr>
<th>Time</th>
<th>Content</th>
<th>Chair</th>
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| 8:30-9:15    | Jean-Dominique Deuschel (Technical University of Berlin)  
Title: Isomorphism Theorems For Ginzburg-Landau Fields                                                                                     | Jürgen Angst                |
| 9:15-10:00   | Jinqiao Duan (Illinois Institute of Technology, Chicago & HUST, Wuhan)  
Title: Geometrical Methods for Stochastic Dynamics                                                                                     | Jinqiao Duan                |
| 10:00-10:15  | Tea Break                                                                                                                                                                                              |                              |
| 10:15-11:00  | Jürgen Angst (University of Rennes 1)  
Title: On the long time behavior of relativistic diffusions                                                                                   | Jinqiao Duan                |
| 11:00-11:45  | Rama Cont (Oxford and Imperial College London)  
Title: TBA                                                                                                                                     |                              |
| Lunch        |                                                                                                                                                                                                        |                              |
| 14:00-14:45  | Naotaka Kajino (Kobe University, Japan)  
Title: The Laplacian on some round Sierpiński carpets and Weyl's asymptotics for its eigenvalues                                              | Xin Chen                    |
| 14:45-15:30  | Francesco Carlo De Vecchi (University of Bonn)  
Title: Gauge symmetries of semimartingales                                                                                                   |                              |
| 15:30-15:45  | Tea Break                                                                                                                                                                                              |                              |
| 15:45-16:30  | Xin Chen (Shanghai Jiao Tong University)  
Title: Random conductance models with stable-like jumps                                                                                      | Naotaka Kajino              |
| 16:30-17:15  | Ionel Popescu (Georgia Institute of Technology)  
Title: Free Functional Inequalities on the Circle                                                                                           |                              |
<p>| 18:30        | Banquet                                                                                                                                                                                               |                              |</p>
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<th>Time</th>
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<tr>
<td>8:30-9:15</td>
<td>Martin Hairer (Imperial College London)</td>
<td>Jia-an Yan</td>
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<td></td>
<td>Title: Stochastic quantization of Yang-Mills</td>
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<td>9:15-10:00</td>
<td>Xicheng Zhang (Wuhan University)</td>
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<td>Title: Singular Brownian diffusion processes</td>
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<td>10:00-10:30</td>
<td>Tea Break and Group Photo</td>
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<tr>
<td>10:30-11:15</td>
<td>Jonas Tölle (Augsburg University, Germany)</td>
<td>Xicheng Zhang</td>
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<td>Title: Stochastic nonlinear PDEs with singular drift and gradient noise</td>
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<td>11:15-12:00</td>
<td>Thomas Cass (Imperial College London)</td>
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<td>Title: Generalisations of the Ito-Stratonovich conversion formula using</td>
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<td>rough paths</td>
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<td>Lunch</td>
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<td>14:00-14:45</td>
<td>Fraydoun Rezakhanlou (University of California, Berkeley)</td>
<td>Robert Weston Neel</td>
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<td>Title: Hamilton-Jacobi PDE and Hamiltonian ODE: A Tale of Two</td>
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<td>Homogenization</td>
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<td>14:45-15:30</td>
<td>Massimiliano Gubinelli (University of Bonn, Germany)</td>
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<td>Title: Gamma convergence and Euclidean QFT</td>
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<td>15:30-15:45</td>
<td>Tea Break</td>
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<tr>
<td>15:45-16:30</td>
<td>Robert Weston Neel (Lehigh University)</td>
<td>Massimiliano Gubinelli</td>
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<td>Title: Geometric and Martin Boundaries on Cartan-Hadamard Manifolds</td>
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<td>16:30-17:15</td>
<td>Xuemei Li (Imperial College London)</td>
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<td></td>
<td>Title: Uniform CLT under Hörmander’s conditions</td>
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<tr>
<td>18:30</td>
<td>Dinner</td>
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III. Titles and Abstracts

1. Mini Courses

Yvain Bruned
Imperial College London

**Title:** Renormalisation in Regularity Structures

**Abstract:** Renormalisation plays an important role in the theory of regularity structures. It appears in different forms. One renormalisation is used for recentering new monomials around a certain point and the other renormalises these ill-defined monomials. They are both described by two Hopf algebras. We will present in this mini-course examples and their constructions.

Martin Hairer
Imperial College London

**Title:** Introduction to Regularity Structure

**Abstract:** A 10 hours mini course on Regularity Structures.

Thomas James Holding
Imperial College London

**Title:** Non-local regularity structures

**Abstract:** The theory of regularity structures has been very successful in solving stochastic partial differential equations of parabolic or elliptic type whose Greens functions are smooth away from the origin. However, in many cases, for example in stochastic wave equations, the Greens function is singular on a much larger set which means the usual theory does not apply. This course will introduce a non-local variant of regularity structures which allows these problems to be tackled.

Konstantin Matetski
University of Toronto

**Title:** The KPZ fixed point

**Abstract:** The KPZ universality class contains one dimensional growth models, directed random polymers, stochastic Hamilton-Jacobi equations. It is characterized by unusual scale of fluctuations, some of which come from random matrix theory, and which are model independent but do depend on the initial data. The physical explanation is that in the space of Markov processes, these models are all being rescaled to a universal fixed point. This scaling invariant fixed point was completely unknown until this year, when we managed to compute it in joint work with J. Quastel and D. Remenik. In the talk I will describe it, as well as how it was obtained by solving a special model in the KPZ universality class called TASEP.
Hao Shen
Columbia University/University of Wisconsin - Madison

Title: TBA
Abstract: TBA

Weijun Xu
New York University Shanghai

Title: Weak universalities
Abstract: One of the motivations to study singular stochastic PDEs is that many of them are expected to be universal objects in crossover regimes in their respective universality classes. We will use the KPZ equation and the three dimensional stochastic quantisation equation as two primary examples, and demonstrate how they naturally arise as the universal limits from a large class of interface growth models and phase coexistence models.

Xiangchan Zhu
Beijing Jiao Tong University

Title: Paracontrolled Distributions
Abstract: In this lecture I will give an introduction for the paracontrolled distribution method which can be used to solve a large class of singular SPDEs introduced by Gubinelli-Imkeller-Perkowski. The idea comes from paradifferential calculus and on ideas from the theory of controlled rough paths. As an application, I will use it to give a meaning to the famous KPZ equations.
2. Workshop

Jürgen Angst
University of Rennes 1

Title: On the long time behavior of relativistic diffusions
Abstract: We will describe the long time asymptotic behavior of relativistic diffusions i.e. diffusion processes with values in Lorentzian manifolds whose law is equivariant under the action of isometries. In particular, we will compare the random compactification of the base manifold given by the exit points of the process, to purely geometric compactifications such as the conformal or causal boundaries.

Marc Arnaudon
University of Bordeaux

Title: A duality formula and a particle Gibbs sampler for continuous time Feynman-Kac measures on path spaces
Abstract: Continuous time Feynman-Kac measures on path spaces are central in applied probability, partial differential equation theory, as well as in quantum physics. I will present a new duality formula between normalized Feynman-Kac distribution and their mean field particle interpretations. Among others, this formula will allow to design a reversible particle Gibbs-Glauber sampler for continuous time Feynman-Kac integration on path spaces. This result extends the particle Gibbs samplers introduced by Andrieu-Doucet-Holenstein in the context of discrete generation models to continuous time Feynman-Kac models and their interacting jump particle interpretations. I will also provide new propagation of chaos estimates for continuous time genealogical tree based particle models with respect to the time horizon and the size of the systems. These results allow to obtain sharp quantitative estimates of the convergence rate to equilibrium of particle Gibbs-Glauber samplers.

Atsushi Atsuji
Keio University, Japan

Title: Leafwise Brownian motions and some function theoretic properties of laminations
Abstract: In 1983 L.Garnett introduced the notion of leafwise Brownian motions and harmonic measures on foliated manifolds. We construct a diffusion process on general foliated spaces, called laminations, whose invariant measure is harmonic. We will see some properties of the diffusion process and discuss some function theoretic properties such as Liouville properties and Picard type theorems of leafwise functions using stochastic calculus of the diffusion process.

Thomas Cass
Imperial College London

**Title:** Generalisations of the Ito-Stratonovich conversion formula using rough paths

**Abstract:** Lyons' theory of rough paths allows one to solve stochastic differential equations driven by a Gaussian processes $X$ under certain conditions on the covariance function. The rough integral of these solutions against $X$ again exist, and a natural question is to find a closed-form conversion formula between this rough integral and the Skorohod integral of the solution which generalises the classical Stratonovich-Ito conversion formula. Previous works in the literature assumes the integrand to be the gradient of a smooth function of $X$; our formula again recovers these results as special cases. Joint work with Nengli Lim.

**Xin Chen**
Shanghai Jiao Tong University

**Title:** Random conductance models with stable-like jumps

**Abstract:** We study the quenched invariance principle and two-sided heat kernel estimates for random conductance models with long range jumps on $\mathbb{Z}^d$, where the transition probability from $x$ to $y$ is in average comparable to $|x-y|^{d-\alpha}$ with $\alpha \in (0,2]$ and the associated conductances are not uniformly elliptic. Under some moment conditions on the conductance, we prove that the scaling limit of the Markov process is a symmetric $\alpha$-stable Lévy process on $\mathbb{R}^d$. We also prove that (elliptic) Harnack inequalities do not hold in the present setting. Our results could be applied to general discrete metric measure space. The talk is based on a joint paper with Takashi Kumagai and Jian Wang.

**Zhenqing Chen**
University of Washington

**Title:** A priori Holder estimate for non-local parabolic SPDE

**Abstract:** In this talk, I will present an $L^\infty$ estimate for non-local parabolic SPDE with measurable jumping kernel, using an improved version of stochastic De Giorgi iteration. Then a priori Holder estimates will be given for solutions of these SPDEs. Joint work with Zhenan Wang.

**Rama Cont**
Oxford and Imperial College London

**Title:** TBA

**Abstract:** TBA

**Jean-Dominique Deuschel**
Technical University of Berlin

**Title:** Isomorphism Theorems For Ginzburg-Landau Fields

**Abstract:** We derive certain identities in law relating functionals of convex gradient
fields to the local times of corresponding random walks in the associated Helffer-Sjöstrand representation. When restricting these identities to Gaussian measures, one recovers classical isomorphism theorems due to Dynkin, Ray-Knight and Le Jan. We apply these results to prove the existence of mass gaps for a class of anharmonic models with suitable single-spin distribution, thus extending results of Brydges, Fröhlich and Spencer. This is a joint work with P.-F. Rodriguez.

Nicolas Peter Dirr  
Cardiff University, UK

Title: Gradient flows and stochastic particle systems
Abstract: We explore the connections between macroscopic gradient flows and microscopic particle system and how they can be exploited for numerical methods.

Federica Dragoni  
Cardiff University, UK

Title: Stochastic Homogenization in Carnot Groups  
Abstract: We present recent homogenization results for Hamilton-Jacobi equations which are not coercive, i.e. classical techniques cannot be applied. Instead, the relation of these Hamiltonians with the geometry of Carnot groups is used.

JinQiao Duan  
Department of Applied Mathematics, Illinois Institute of Technology, Chicago & Center for Mathematical Sciences, Huazhong Univ of Sci and Tech, Wuhan

Title: Geometrical Methods for Stochastic Dynamics  
Abstract: Dynamical systems arising in engineering and science are often subject to random fluctuations. The noisy fluctuations may be Gaussian or non-Gaussian, which are modeled by Brownian motion or $\alpha$-stable Levy motion, respectively. Non-Gaussianity of the noise manifests as nonlocality at a “macroscopic” level. Stochastic dynamical systems with non-Gaussian noise (modeled by $\alpha$-stable Levy motion) have attracted a lot of attention recently. The non-Gaussianity index $\alpha$ is a significant indicator for various dynamical behaviors.

The speaker will overview recent advances in geometrical methods for stochastic dynamical systems, including random invariant sets, random invariant manifolds, stochastic bifurcation, mean exit time, escape probability, tipping time, most probable orbits, and transition pathways between metastable states.

David Elworthy  
Mathematics Institute, University of Warwick, UK

Title: Towards higher order derivative formulae  
Abstract: I will discuss methods of obtaining formulae for higher order derivatives of
heat semigroups based on techniques from earlier work with XueMei Li and Yves LeJan. A basic example will be for the heat semigroup on spheres. I will aim to show that the problem is both more complicated and more interesting than might first be expected.

Martin Grothaus  
Technical University of Kaiserslautern, Germany

**Title:** Approximation of the stochastic heat equation with sticky reflected boundary condition  
**Abstract:** In this talk we study the stochastic heat equation with sticky reflected boundary condition. Dirichlet form techniques are used to construct its solution. The obtained process already for some time is conjectured to be the scaling limit of the dynamical wetting model, also known as Ginzburg-Landau dynamics with pinning and reflection competing on the boundary. In this talk it is planned to discuss also about the recent progress on this problem.

Massimiliano Gubinelli  
University of Bonn, Germany

**Title:** Gamma convergence and Euclidean QFT  
**Abstract:** I will describe a new approach to prove existence of the Euclidean $\Phi^4_3$ measure in a periodic domain using variational methods and estimates inspired by the theory of singular SPDEs. The variational problem is formally related to an Hamilton-Jacobi equation first introduced by Wilson in the context of the renormalization group method.

Ben Hambly  
Maths Institute, University of Oxford

**Title:** SPDEs and systemic risk  
**Abstract:** We consider a simple model for systemic risk in a financial market. The individual financial institutions are assumed to be connected in such a way that losses in one institution affect others. By taking a large portfolio limit in the market we derive stochastic McKean-Vlasov equations that describe the financial system as a whole. The effect of the losses introduces feedback effects and the model can capture systemic events as breakdowns in the solutions to these equations.

Martin Hairer  
Imperial College London

**Title:** Stochastic quantization of Yang-Mills  
**Abstract:** We construct the natural dynamic associated to the Yang-Mills measure in dimension 3. In particular, we show that among the many notions of solution provided by the theory of regularity structures, exactly one yields a gauge-equivariant process.
Title: The Laplacian on some round Sierpiński carpets and Weyl's asymptotics for its eigenvalues

Abstract: The purpose of this talk is to present the speaker's recent research in progress on the construction of a "canonical" Laplacian on round Sierpiński carpets invariant with respect to certain Kleinian groups (i.e., discrete groups of Möbius transformations on $\hat{\mathbb{C}}:=\mathbb{C}\cup\{\infty\}$) and on Weyl's asymptotics for its eigenvalues. Here a "round Sierpiński carpet" refers to a subset of $\hat{\mathbb{C}}$ homeomorphic to the standard Sierpiński carpet, such that its complement in $\hat{\mathbb{C}}$ consists of disjoint open disks in $\hat{\mathbb{C}}$.

The construction of the Laplacian is based on the speaker's preceding study of the simplest case of the "Apollonian gasket", the compact fractal subset of $\mathbb{C}$ obtained from an ideal triangle (a triangle formed by mutually tangent three circles) by repeating indefinitely the process of removing the interior of the inner tangent circles of the ideal triangles. On this fractal, Teplyaev (2004) had constructed a canonical Dirichlet form as one with respect to which the coordinate functions on the gasket are harmonic, and the author later proved its uniqueness and discovered an explicit expression of it in terms of the circle packing structure of the gasket.

The expression of the Dirichlet form obtained for the Apollonian gasket in fact makes sense on round Sierpiński carpets and defines (a candidate of) a "canonical" Laplacian on such fractals. When the round Sierpiński carpet is the limit set (i.e., the minimum invariant non-empty compact set) of a certain class of Kleinian groups, some explicit combinatorial structure of the fractal is known and makes it possible to prove Weyl's asymptotic formula for the eigenvalues of this Laplacian, which is of the same form as the circle-counting asymptotic formula by Oh and Shah [Invent. Math. 187 (2012), 1--35].

The difficulty in the case of a round Sierpiński carpet is that, since it is infinitely ramified, i.e., the cells in its cellular decomposition intersect on infinite sets, it is highly non-trivial to show that the principal order term of the eigenvalue asymptotics is not affected by the cellular decomposition, namely by assigning the Dirichlet boundary condition on the boundary of the cells.

Title: Functional central limit theorems for non-symmetric random walks on

- 14 -
nilpotent covering graphs

Abstract: The long time asymptotics for random walks on infinite graphs is a principal topic in both geometry and probability theory. A covering graph of a finite graph with a nilpotent covering transformation group is called a nilpotent covering graph, regarded as a generalization of a crystal lattice or the Cayley graph of a finite generated group of polynomial growth.

In this talk, we discuss non-symmetric random walks on nilpotent covering graphs from a view point of the theory of discrete geometric analysis developed by Kotani and Sunada, and give functional central limit theorems for them. We also mention a relationship between the limiting diffusions and distorted Brownian rough paths.

This talk is based on joint work with Satoshi Ishiwata (Yamagata University) and Ryuya Namba (Okayama University).

Yuri Kifer
Hebrew University, Jerusalem

Title: Geometric law for multiple returns until a hazard
Abstract: For a $\psi$-mixing stationary process we consider the number of multiple returns to a set until the moment (which we call a hazard) when the first multiple return to another set takes place. It turns out that if probabilities to arrive to these sets are of the same order then the above number has asymptotically a geometric distribution. Similar results are obtained in the dynamical systems setup considering $\psi$-mixing shifts on the sequence space. The work is motivated by the research on single and multiple returns to shrinking sets, as well as by the research on open systems studying their behavior until an exit through a "hole". The work is joint with my student A.Rapaport.

Seiichiro Kusuoka
Okayama University

Title: The invariant measure and flow associated to the Phi4-quantum field model on the three-dimensional torus
Abstract: We consider the invariant measure and flow of the Phi4-model on the three-dimensional torus, which appears in the quantum field theory. By virtue of Hairer's breakthrough, such a nonlinear stochastic partial differential equation became solvable and is studied as a hot topic. In the talk, we also apply Hairer's reconstruction of equations and directly construct the global solution and the invariant measure by using the invariant measures of approximation equations and the technique of solving the nonlinear dissipative parabolic equations.

Thierry Lévy
Sorbonne Université

Title: Quantum spanning forests
Abstract: I will report on a work in progress with Adrien Kassel about an extension of Kirchhoff’s matrix-tree theorem and determinantal point processes to the framework of vector bundles over graphs.

Xuemei Li
Imperial College London

Title: Uniform CLT under Hörmander’s conditions

Abstract: We discuss CLT for a family of operators satisfying Hörmander’s conditions, and the uniform rate of convergence.

Kening Lu
Brigham Young University

Title: TBA

Abstract: TBA

Sonia Mazzucchi
University of Trento, Italy

Title: Generalized Feynman-Kac formulae

Abstract: Generalized Feynman-Kac formulae, i.e. probabilistic representations for the solution of PDEs that do not satisfy maximum principle, can be constructed by replacing the “traditional” concept of Lebesgue integral with respect to a $\sigma$-additive bounded measure with the more general concept of linear functional on a suitable space of “integrable functions". In the present talk I shall present the technical difficulties as well as some possible solutions of this problem, showing that the mathematical theory of Feynman path integrals has a wider scope and can be applied to the construction of integral representation for the solution of $N$-order heat-type equations.

Robert Weston Neel
Department of Mathematics, Lehigh University

Title: Geometric and Martin Boundaries on Cartan-Hadamard Manifolds

Abstract: We recall results on the solvability of the Dirichlet problem at infinity and the identification of the geometric and Martin boundaries for Cartan-Hadamard manifolds, by both stochastic and non-stochastic methods. The situation is rather different in the two-dimensional and higher-dimensional cases. In the two-dimensional case, by studying the behavior of Brownian motion, we show that any upper radial curvature bound yielding transience also yields solvability of the Dirichlet problem at infinity, and we indicate what this implies for the relationship between the geometric boundary and the Martin boundary. Further, if the curvature is bounded from below, we show that any upper radial curvature bound giving
transience also gives that the Martin boundary is homeomorphic to the sphere at infinity.

Hirofumi Osada
Kyushu University, Japan

Title: Infinite-dimensional stochastic differential equations with symmetry
Abstract: I talk a method to solve infinite-dimensional stochastic differential equations (ISDE) with symmetry. This class of ISDE naturally appears in statistical physics and describes infinite particle systems.

Ionel Popescu
Georgia Institute of Technology

Title: Free Functional Inequalitites on the Circle
Abstract: We will present some free functional inequalities on the circle. For instance, we will present a transportation inequality, a Log-Sobolev type and HWI. All these inequalities have a little different form from the classical case which seems to be due to the fact that the circle acts on itself. This begs several questions in the classical counterparts which have not been investigated yet.

Fraydoun Rezakhanlou
University of California, Berkeley

Title: Hamilton-Jacobi PDE and Hamiltonian ODE: A Tale of Two Homogenization
Abstract: A Hamilton-Jacobi PDE is closely related to a Hamiltonian ODE. If the Hamiltonian function is random and translation invariant with respect to space shifts, then the long time/space asymptotic of solutions can be studied and is related to the homogenization phenomenon. When the Hamiltonian function is convex in the momentum variable, the homogenization question for the Hamiltonian-Jacobi PDE and the corresponding Hamiltonian ODE is more or less equivalent. This is no longer the case when the Hamiltonian function is not convex. The homogenization question for Hamiltonian ODE only when the Hamiltonian function is periodic in spatial variable has been established with the help of techniques from symplectic geometry (Viterbo 2007). It remains open in the stochastic case.

Michael Röckner
Bielefeld University and AMSS. CAS

Title: Nonlinear Fokker-Planck-Kolmogorov equations and distribution dependent SDE
Abstract: By Ito’s formula the time marginals of a solution to a distribution dependent SDE solve a nonlinear Fokker-Planck-Kolmogorov equation. This talk is about the converse: we present a general technique how to identify a solution to a nonlinear Fokker-Planck-Kolmogorov equation consisting of probability densities as
the time marginals of a solution to a distribution dependent SDE. We apply this to the special case of a porous media equation perturbed by the divergence of a vector field depending non-linearly on the solution. More precisely, we construct a generalized entropic solution $u$ to this equation and apply the above general technique to find the corresponding distribution dependent SDE which has a weak solution with marginals given by $u$. We thus gain a probabilistic representation of $u$.

(joint work with Viorel Barbu, Romanian Academy of Sciences, Iasi)

Reference: arXiv:1801.10510

Barbara Rüdiger
University Wuppertal

Title: The Boltzmann Process
Abstract: We derive a McKean-Vlasov equation for which the solution is distributed according to the Boltzmann equation. We call its solution the Boltzmann process. This means that the Kolmogorov equation associated to a Boltzmann process is the Boltzmann equation. We analyse smooth conditions under which the solution of the Boltzmann equation guarantees the existence of a Boltzmann process. This is based on a joint work with S. Albeverio, P. Sundar.

Paulo Régis Caron Ruffino
University of Campinas, Brazil

Title: Geodesic jumps in non-continuous SDE: applications to an averaging principle on foliated space
Abstract: Semimartingales with jumps have been treated among others, by Kurtz, Pardoux and Protter, 1995, using the so called Marcus approach for jumps. Marcus interpretation describes the jumps as following an artificial deterministic flow of a vector field along a hidden time. Here we propose jumps of cadlag trajectories along geodesics, hence depending only on the point where the jumps start at. We get a generalized Itô-Kunita decomposition of the corresponding flow of local diffeomorphism and apply this approach to decomposition of flows and averaging along foliated manifolds.

Hao Shen
Columbia University/University of Wisconsin - Madison

Title: TBA
Abstract: TBA

Alain-Sol Sznitman
ETH Zurich
Title: On macroscopic holes in some dependent percolation models
Abstract: We consider on \( \mathbb{Z}^d \), with \( d \geq 3 \), the vacant set of random interlacements in the strongly percolative regime, the vacant set of the simple random walk, and the excursion set of the Gaussian free field in the strongly percolative regime. We present asymptotic upper and lower exponential bounds for the large deviation probability that the adequately thickened component of the boundary of a large box centered at the origin in the respective vacant sets or excursion set leaves in the box a macroscopic volume in its complement, as well as some geometric controls on the shape of the left-out volume.

Anton Thalmaier
University of Luxembourg

Title: Characterization of Ricci curvature and Ricci flow by Brownian motion
Abstract: We present recent work on describing Ricci curvature and Ricci flow in terms of functional inequalities for heat semigroups on manifolds. The inequalities are strong enough to characterize in particular Einstein manifolds and Ricci solitons. The talk includes extensions of these methods to geometric flows on manifolds, as well as to the path space of Riemannian manifolds evolving under a geometric flow.

James Thompson
University of Luxembourg

Title: Quantitative gradient estimates by Bismut formulae
Abstract: For a \( C^2 \) function \( u \) and an elliptic operator \( L \) on a smooth manifold, we will use stochastic analysis to prove a quantitative local estimate for the derivative \( du \) in terms of local bounds on \( u \) and \( Lu \). This extends the recent work of G"{u}neysu and Pigola [1], in which analytic methods were used. An integral version of our estimate can be used to derive a zero-mean value condition for \( \Delta u \). Extensions to differential forms and manifolds with a boundary are also possible.

Although there is a long history of using stochastic analysis to study solutions to partial differential equations with various boundary conditions, it is nonetheless surprising that explicit estimates of this type can be obtained from the stochastic analysis of Brownian motion in such a simple and versatile way.

This is joint work with Prof. Dr. A. Thalmaier and Dr. L.-J. Cheng. Our main results are covered by the following two preprints: arXiv:1707.07121 and arXiv:1803.08844.


Jonas Tölle
Augsburg University, Germany
Title: Stochastic nonlinear PDEs with singular drift and gradient noise

Abstract: We shall discuss well-posedness results for stochastic nonlinear parabolic PDEs with singular drift and gradient Stratonovich noise with coefficients that may depend on the spatial variable. The drift term is given by a realization of a $p$-Laplace-type operator (for the singular cases $1 \leq p \leq 2$), including also the more general case of non-homogeneous or multi-valued nonlinearities. For initial data in $L^2$, we prove the unique existence of a continuous process solving the SPDE in the sense of stochastic variational inequalities. The results are based on geometric conditions on the spatial domain and its boundary symmetries being related to the Itô-Stratonovich-corrector of the gradient noise. By imposing a curvature-dimension condition as well as a defective commutation condition, we obtain the higher order a priori estimates that allow us to pass to the limit in the approximation of the solution.

The results are partially based on a joint work with Ioana Ciotir (Normandie Université, INSA Rouen).

References:

Nizar Touzi
Ecole Polytechnique, France

Title: Branching particles representation for nonlinear PDEs

Abstract: We provide a probabilistic representations of the solution of some semilinear hyperbolic and high-order PDEs based on branching diffusions. These representations pave the way for a Monte-Carlo approximation of the solution, thus bypassing the curse of dimensionality. We illustrate the numerical implications in the context of some popular PDEs in physics such as nonlinear Klein-Gordon equation, a
simplified scalar version of the Yang-Mills equation, a fourth-order nonlinear beam equation and the Gross-Pitaevskii PDE as an example of nonlinear Schrödinger equations.

Stefania Ugolini  
Department of Mathematics, University of Milan

**Title:** A stochastic approach to Bose-Einstein Condensation  
**Abstract:** A well-posed probabilistic way of looking at the Bose-Einstein condensation consists in rigorously associating a N-dimensional diffusion process to the ground state eigenfunction of the N-body Hamiltonian through Nelson map. We describe some probability measures convergence problems related to the Gross-Pitaevskii scaling limit of infinite particles. We discuss the entropy chaos for the symmetric probability law of the N interacting diffusion system and the weak convergence on the path space of the one particle probability law to the probability measure uniquely associated with the minimizer of the nonlinear Gross-Pitaevskii functional. The talk is based on a joint work with Sergio Albeverio and Francesco Carlo De Vecchi.

Francesco Carlo De Vecchi  
University of Bonn

**Title:** Gauge symmetries of semimartingales  
**Abstract:** In this talk we introduce the concept of gauge symmetry group of a general semimartingale with jumps. This concept was originally proposed for explaining the idea of weak symmetries of Brownian-motion-driven SDEs and it is exemplified by the group of random rotations of a Brownian. Then we explain some practical methods, exploiting the characteristic triplet of a semimartingale, for verifying the presence of gauge symmetries for specific semimartingales. Finally we use this notion to introduce a definition of weak symmetry for SDEs driven by general semimartingales with jumps and we show some concrete examples of weak symmetric SDEs.  

The talk is based on the paper "Symmetries and invariance properties of stochastic differential equations driven by semimartingales with jumps" (arXiv:1708.01764) written in collaboration with Sergio Albeverio, Paola Morando and Stefania Ugolini.

Fengyu Wang  
Tianjin University

**Title:** Estimates of Invariant Probability Measures for Singular SDEs  
**Abstract:** In terms of a nice reference probability measure, integrability conditions on the path-dependent drift are presented for (infinite-dimensional) degenerate PDEs to have regular positive solutions. To this end, the corresponding stochastic (partial) differential equations are proved to possess the weak existence and uniqueness of solutions, as well as the existence, uniqueness and entropy estimates
of invariant probability measures. When the reference measure satisfies the log-Sobolev inequality, Sobolev estimates are derived for the density of invariant probability measures. Some results are new even for non-degenerate SDEs with path-independent drifts. The main results are applied to nonlinear functional SPDEs and degenerate functional SDEs/SPDEs.

**Bin Xie**  
Shinshu University

**Title:** On the space-time white noise driven SPDE with reflection  
**Abstract:** The stochastic partial differential equation with reflection is one kind of random parabolic obstacle problems, which is also very important in applications. In this talk, we will discuss the dimension-free Harnack inequality for the Markov semigroup associated with the reflected SPDE driven by multiplicative space-time white noise. In addition, we will also consider the hypercontractive property relative to the reflected SPDE driven by additive noise.

**Tusheng Zhang**  
University of Manchester, UK

**Title:** Small time asymptotics of Brownian motion with singular drifts  
**Abstract:** In this talk I will present a recent result on small time large deviations and a varadhan type small time asymptotics for Brownian motion with measure drift.

**Xicheng Zhang**  
Wuhan University

**Title:** Singular Brownian diffusion processes  
**Abstract:** In this talk we survey some recent progress about the SDEs with distributional drifts and singular measured-valued drifts. In particular, we show the well-posedness of martingale solutions or weak solutions, and obtain sharp two-sided and gradient estimates of the heat kernel associated to the above SDE. In one dimensional case, we also discuss the strong uniqueness of the solutions. Moreover, we also study the ergodicity and global regularity of the invariant measures of the associated semigroup under some dissipative assumptions.
IV. Participants

Speakers of Mini Courses:

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<th>No</th>
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<td>1.</td>
<td>Yvain Bruned</td>
<td>Imperial College London</td>
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<td>Martin Hairer</td>
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<td>Thomas James Holding</td>
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<td>Konstantin Matetski</td>
<td>University of Toronto</td>
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<td>Hao Shen</td>
<td>Columbia University / University of Wisconsin-Madison</td>
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<td>Weijun Xu</td>
<td>New York University Shanghai</td>
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<td>Xiangchan Zhu</td>
<td>Beijing Jiao Tong University</td>
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Speakers of Workshop:

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<td>Jürgen Angst</td>
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<td>Marc Arnaudon</td>
<td>University of Bordeaux</td>
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<td>Atsushi Atsuji</td>
<td>Keio University, Japan</td>
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<td>Thomas Cass</td>
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<td>Xin Chen</td>
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<td>Jean-Dominique Deuschel</td>
<td>Technical University of Berlin</td>
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<td>Federica Dragoni</td>
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<td>Jinqiao Duan</td>
<td>Illinois Institute of Technology, Chicago &amp; Huazhong Univ of Sci and Tech, Wuhan</td>
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<td>David Elworthy</td>
<td>Mathematics Institute, University of Warwick</td>
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<td>Martin Grothaus</td>
<td>Technical University of Kaiserslautern, Germany</td>
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<td>Naotaka Kajino</td>
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<td>University of California, Berkeley</td>
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<td>Bielefeld University and AMSS (CAS)</td>
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<td>Wuhan university</td>
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V. Transportation

There are mainly three ways to AMSS or recommended hotels from the Beijing Capital International Airport (BCIA):

1. Taxi (highly recommended)

The most convenient way to AMSS or recommended hotels is by taxi. Normally it costs about CNY 100 (roughly USD 15 or GBP 10) from BCIA to arrive at AMSS or hotels. You can easily find taxi stops by following the taxi signs at the Airport. Please present the map to Taxi driver who should then be able to take you to the destination easily.

Note: The minimum charge for taxi is CNY 10 covering the first 3km, and the rest will be charged at CNY 2 per km together with CNY 3 extra fuel charge. Additional 50% of the total fee will be charged as an allowance for the return trip if the single trip has exceeded 15km. The minimum charge will be RMB 11 from 23:00pm to 5:00am, and there is also a rise of 20% per kilometer. You are also required to pay CNY 10 for the toll at BCIA.

Please pay according to the amount displayed on the fare calculator on the taxi plus some possible extra fees mentioned above. Please always ask for the taxi receipt for your record (in case you lose some personal stuff on the taxi, we can find the taxi through the information on the receipt). Tips are not necessary.

2. Airport Express Railway & Subway (Recommended)

You can also take Airport Express Railway and then transfer to Subway. This is a good choice especially during the peaking hour. The closest Subway station to AMSS or recommended hotels is Zhichunli (知春里). More precisely, first take Airport Express Railway at Terminals 2 or 3 to Sanyuanqiao (三元桥), and transfer to Subway Line 10 to Zhichunli (知春里) station. Then you can choose to walk to
AMSS or hotels. The total fare for a single trip is about CNY 35, including CNY 25 for Airport Express Railway and CNY 6 for Subway.

A gentle reminder: please keep your train/subway ticket during your travel. When you get off the train, you need return your ticket to pass through the Subway exit control.

3. Airport Shuttle

Charge Standard (Local buses) : CNY 30 per single trip.

Tickets Offices (from BCIA) :

Terminal 1: Gate No. 7 (inside) on F1 (the First Floor)

Terminal 2: Gate No. 9 to No.11 (outside) on F1

Terminal 3: Exit of Zone A, opposite to the exit of Zone C on F2; next to Gate5, 7&11 on F1

Timetable: The first bus leaves the airport at 6:50am, and the second one leaves at 7:00am, then during 7:00~24:00 there will be one bus for every 20 minutes

Routes: Line 5 : BCIA (T3/T2/T1) ----> Zhongguancun (中关村) (Fourth Bridge (四号桥)). Then you can walk to AMSS or recommended hotels or take a taxi (costs CNY 10) about 1.5km.