Heat kernel asymptotics at the cut locus

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Time Tuesday, July 21, 2015 at 16:00 Place Campus Kirchberg, room B04

We discuss a technique, going back to work of Molchanov, for determining the small-time asymptotics of the heat kernel (equivalently, the large deviations of the corresponding diffusion) at the cut locus of a sub-Riemannian manifold (valid away from any abnormal geodesics). We relate the leading term of the expansion to the structure of the cut locus, especially to conjugacy, and explain how this can be used to find general bounds as well as to compute specific examples, in both the Riemannian and sub-Riemannian contexts. We also show how this approach leads to restrictions on the types of singularities of the exponential map that can, generically, occur along minimal geodesics. Further, we discuss the asymptotics for the gradient and Hessian of the logarithm of the heat kernel on a Riemannian manifold, giving a characterization of the cut locus in terms of the behavior of the log-Hessian. In particular, the leading term in the expansion of the log-Hessian comes from the variance "of the minimal geodesics" with respect to the small-time Brownian bridge. Much of this work is joint with Davide Barilari, Ugo Boscain, and Grégoire Charlot.