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Malliavin, Paul; Thalmaier, Anton

Stochastic calculus of variations in mathematical finance. (English)

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This textbook is devoted to the stochastic calculus of variations (Malliavin calculus) and its applications in Mathematical Finance. The Malliavin calculus is an infinite-dimensional differential calculus for stochastic processes with or without continuous paths. Its main objective is the study of regularity properties of probability distributions.

The book under review demonstrates the power and versatility of the Malliavin calculus in a variety of problems arising in Mathematical Finance. Despite being mathematically demanding, it is directed not only towards researchers in mathematics, but also to practitioners through the choice of topics presented, which include practical questions such as price sensitivities as well as numerical problems, e.g. Monte Carlo simulation of conditional expectations. The presentation of the mathematical theory is rather compact but still self-contained and aims at an application oriented point of view, guiding the reader through a selection of problems that are close to applications on the one hand, but require deep understanding of Malliavin's calculus on the other hand.

The book will certainly address in the first place researchers in mathematical finance. It can however be recommended to a much wider public in mathematics beyond probability: all those who want to see a very lucid account of an example in which sophisticated concepts of variational calculus on the borderline between probability and analysis meet and show surprising efficiency in very practical problems of financial engineering.

The book consists of eight chapters and three appendices, and is equipped with a comprehensive and rather up-to-date list of references. In the first chapter the basics of Malliavin calculus are recalled, with applicability to financial problems in mind.

Chapter two starts with a classical problem in option pricing theory, the sensitivities of option prices with respect to model parameters. These sensitivities (Greeks) are important to traders, since they express the risk in trading options. Using Malliavin calculus, the authors derive formulas for sensitivities that are suitable for practical purposes, e.g. Monte Carlo methods.

Chapter 3 on market equilibrium and price-volatility feedback rates is based on a paper by the authors and their co-workers. The price-volatility feedback rate describes the market's stability and is a liquidity index of the market. It is given by the logarithmic derivative of the stock price sensitivity, measured (rescaled) in units of the volatility. Negative feedback rates correspond to a stable market situation, while positive feedback rates indicate instability. This is illustrated in a practical situation in Appendix C for the IBM stock, and is underpinned by the theoretical result that – modulo regularity conditions – a negative feedback rate implies that the market is ergodic.

In practice one often faces the problem of calculating conditional expectations, e.g.

prices of American options, a task that is highly non-trivial from a numerical point of view. In Chapter 4, the authors derive expressions for conditional expectations that are suitable for Monte Carlo simulations. Using tools from Malliavin calculus, they develop a theory for multivariate conditioning, and turn the problem of calculating conditional expectations into the problem of calculating a functional expectation of the random variable, a divergence operator and differentiating. These results broaden the scope of Monte Carlo methods in financial applications.
Chapter 5 deals with interest rate models and problems with hedging instabilities. Due to high dimensions of the market models, while variance is low-dimensional on the other side, ellipticity is too strong a condition to work with. Therefore, the authors use hypoellipticity instead, and in particular study a Heath-Jarrow-Morton model for the interest-rate market. They use the pathwise compartment principle, stating that the covariance matrix strictly increases with time. This entails that the norms for different times are not equivalent, so that hedging becomes unstable for certain digital options.
Chapter 6 is devoted to insider trading. An insider is a financial agent who can access more information than the market, which mathematically amounts to an enlargement of the filtration that describes the market's flow of information and leads to problems of anticipative stochastic calculus. The authors address the question of additional utility for an insider investing in the market. It is shown that insider information corresponds to an additional information drift on the underlying price processes. This drift adds to the insider's maximal utility and quantifies the value of having additional information.
The topic of Chapter 7 is the Euler scheme for stochastic differential equations. Its discretization error is expressed as a generalized Watanabe distribution on the Wiener space, which leads to an asymptotic expansion of the price of a digital option depending on a parameter. Moreover, the convergence of the Euler scheme in a strong sense (w.r.t. Sobolev norms on the Wiener space) and in a weak sense is proved.
The last chapter provides an introduction to a stochastic calculus of variations for markets with jumps, where the randomness originates in a compound Poisson driving process. The derivative operator is defined, and an integration-by-parts formula for this situation is proved, which allows for the calculation of Greeks of certain options. The results are applied in order to explicitly calculate mean-variance hedging strategies.

Peter Imkeller (Berlin)

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Classification :
∗60H07 Stochastic calculus of variations and the Malliavin calculus
91B28 Finance etc.
91-02 Research exposition (Social and behavioral sciences)
60H30 Appl. of stochastic analysis
60-02 Research monographs (probability theory)
60G44 Martingales with continuous parameter
91B24 Price theory and market structure