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Mathematics Research Unit
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Exercises from Lecture 4

Riemann Surfaces

Exercise 18: Check that a universal covering is unique up to a homeomorphism.

Exercise 19: Find a universal covering $\widetilde{X}$ of a complex torus $X = \mathbb{C}/\Gamma$. Compute its group of deck transformations.

Exercise 20: Let $\Gamma = \mathbb{Z} + \mathbb{Z} \cdot \tau$, $\tau \in \mathbb{C}$, be a lattice in $\mathbb{C}$. Let n be a natural number and let $\Gamma' = \mathbb{Z} + \mathbb{Z} \cdot (n\tau)$. Put $X = \mathbb{C}/\Gamma$ and $X' = \mathbb{C}/\Gamma'$ and consider the map

$$X \to X', \quad [z] \mapsto [nz].$$

By Exercise 6 it is a holomorphic map of Riemann surfaces. Prove that it is a covering. What is the number of points in the fibres? Compute the group of deck transformations of this covering.

Exercise 21: Show that the map $\mathbb{C} \xrightarrow{\exp} \mathbb{C}^*$ is a covering. Compute its group of deck transformations.