University of Luxembourg
Mathematics Research Unit
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Exercises from Lecture 6

Riemann Surfaces

Exercise 26: Check that the definition of a holomorphic differential form does not depend on the choice of a local coordinate.

Exercise 27: Let X be a Riemann surface and let $z : U \to V$ and $w : U \to W$ be two complex charts of X. Then $w = G \circ z$ for a holomorphic function $G : V \to W$. Show that

$$dw = G'(z) \cdot dz.$$

Exercise 28: Prove that $\Omega_{\hat{\mathbb{C}}}(\hat{\mathbb{C}}) = 0$.
Hint: *Consider an arbitrary $\omega \in \Omega_{\hat{\mathbb{C}}}(\hat{\mathbb{C}})$ in the standard charts of $\hat{\mathbb{C}}$ and apply Exercise 27 to the intersection of the standard chars.*

Exercise 29: Let ω be a meromorphic differential form on a Riemann surface X and let a be a point of X. Check that $\operatorname{ord}_a \omega$ defined in the lecture does not depend on the choice of a local coordinate.