Noncommutative harmonic analysis and representation theory
Luxembourg, June 14-17, 2011

Abstracts

Robert Archbold (University of Aberdeen, UK)

Title: The Brown-Pedersen topological dimension and the real rank of some group C^*-algebras.

Abstract: In this talk, I will begin by describing the Brown-Pedersen topological dimension $\operatorname{top}\dim(A)$, for a type I C^*-algebra A, and its recent use by Larry Brown in determining the real rank and stable rank of CCR C^*-algebras. These methods are then applied to some group C^*-algebras. An important first step is to establish the finiteness of $\operatorname{top}\dim(C^*(G))$ for various locally compact groups G. Then the results of Brown can be combined with some representation theory to determine the real rank $\mathrm{RR}(C^*(G))$. For example, if G is an almost connected, nilpotent group then

$$\mathrm{RR}(C^*(G)) = \operatorname{rank}(G/[G,G]) = \operatorname{rank}(G_0/[G_0,G_0]),$$

where G_0 is the connected component of the identity element. This is joint work with E. Kaniuth (Paderborn).

Ali Baklouti (University of Sfax, Tunisia)

Title: On the estimate of the Fourier transform norm of certain Lie groups.

Abstract: I will speak about the Fourier transform of L^p functions $(1 < p \leq 2)$ on some Lie groups. The objective is to get an estimate of its norm and discuss the problem of sharpness. I will focus first on the case of connected nilpotent Lie groups and study then some compact extensions.

Paul Baum (Pennsylvania State University, USA)

Title: Geometric structure in the representation theory of reductive p-adic groups

Abstract: Let G be a reductive p-adic group. Examples are $GL(n,F)$, $SL(n,F)$ where n can be any positive integer and F can be any finite extension of the field Q_p of p-adic numbers. The smooth (or admissible) dual of G is the set of equivalence classes of smooth irreducible representations of G. The representations are on vector spaces over the complex numbers. The smooth dual has one point for each distinct smooth irreducible representation of G. Within the smooth dual there are subsets known as the Bernstein components, and the smooth dual is the disjoint union of the Bernstein components. This talk will explain a conjecture due to Aubert-Baum-Plymen (ABP) which says that each Bernstein component is a complex affine variety. These affine varieties are explicitly identified as certain extended quotients. The infinitesimal character of Bernstein and the L-packets which appear in the local Langlands conjecture are then described from this point of view. Recent results by a number of mathematicians (e.g. V. Heiermann, M. Solleveld) provide positive evidence for ABP.

Bachir Bekka (Université de Rennes, France)

Title: On the ergodic theory of groups of automorphisms of nilmanifolds

Abstract: Let N be a connected nilpotent Lie group and D a lattice in N. Let Γ be a group of automorphisms of N preserving D. Then Γ acts by measure preserving transformations on the nilmanifold N/D. We discuss the spectral decomposition of $L^2(N/D)$ under the associated unitary representation of Γ and give a necessary and sufficient condition under which a spectral gap exists for the Γ action on N/D.

Jean-Louis Clerc (Université Henri Poincaré, Nancy, France)

Title: Conformally invariant trilinear forms and covariant differential operators on the sphere

Abstract: Let $S = S^{n-1}$ be the unit sphere, and let $G = SO_0(1, n)$ be the conformal group of the sphere. For $\lambda \in \mathbb{C}$, let π_λ be the principal series representation of G, realized on $\mathcal{C}^\infty(S)$. Given three complex parameters $\lambda_1, \lambda_2, \lambda_3$, a continuous trilinear form on $\mathcal{C}^\infty(S)\times\mathcal{C}^\infty(S)\times\mathcal{C}^\infty(S)$ is constructed, which is invariant under $\pi_{\lambda_1}\otimes\pi_{\lambda_2}\otimes\pi_{\lambda_3}$. The form is obtained by meromorphic continuation in the λ's, with simple poles along four families of (explicitly defined) planes in $\mathbb{C}^3$. The trilinear form is generically unique (up to a factor). The residues along the planes of poles are computed, with the help of a Bernstein-Sato identity. The residues correspond to singular distributions on $S \times S \times S$, and their expressions involve covariant differential operators and covariant bidifferential operators.

Hidenori Fujiwara (Kinki University, Kyushu, Japan)

Title: Intertwining integrals on completely solvable Lie groups

Abstract: Let $G = \exp \mathfrak{g}$ be a completely solvable Lie group with Lie algebra $\mathfrak{g}$. Let $\mathfrak{h}_1, \mathfrak{h}_2$ be two polarizations at $f \in \mathfrak{g}^*$ satisfying the Pukanszky condition. As usual we consider the unitary character χ_f of $H_j = \exp \mathfrak{h}_j$ $(j = 1, 2)$ defined by $\chi_f(\exp X) = e^{if(X)}$ $(X \in \mathfrak{h}_j)$ and induce $\rho_j = \mathrm{ind}_{H_j}^G \chi_f$. Then, it is well known that these two monomial representations ρ_1, ρ_2 are equivalent. A formal intertwining operator between them is given by an integral whose convergence is problematic. We discuss on this integral.

Piotr M. Hajac (IMPAN, Warsaw, Poland)

Title: The Klein-Podleś bottle as a non-trivial bundle over the quantum real projective space $RPq(2)$

Abstract: We tensor the C^*-algebra of the equatorial Podleś quantum sphere with the algebra of continuous functions on the unit circle, act on the tensor product with the diagonal antipodal $\mathbb{Z}/2$-action, and consider the invariant subalgebra. This gives a $U(1) - C^*$-algebra A with the quantum real projective space C^*-algebra $C(RPq(2))$ as its U(1)-invariant part. Using the identity representation of $U(1)$, we associate with it a finitely generated projective module over $C(RPq(2))$. Combining methods of topology and operator algebras, we prove that this module is not free. This implies that A cannot be a crossed product of $C(RPq(2))$ and the integers. To compute the K-theory of A, we present it as a pullback of two copies of the tensor product of the equatorial sphere C^*-algebra with the algebra of continuous functions on the interval [0,1]. We put these two copies together by the identity and antipodal automorphisms of the quantum sphere applied at 0 and 1, respectively. Hence A can be viewed as defining a non-trivial bundle over a circle with the fibre being the equatorial quantum sphere. Since replacing the quantum sphere with the unit circle and the antipodal action with the complex conjugation would yield the Klein bottle, we call A the C^*-algebra of the Klein-Podleś bottle. (Joint work with P.F. Baum.)

Dominique Manchon (CNRS, Clermont-Ferrand, France)

Title: On Hopf algebras of oriented Feynman graphs

Abstract: We define two coproducts for cycle-free oriented graphs, thus building up two commutative connected graded Hopf algebras, such that one is a comodule-coalgebra on the other, thus generalizing the result obtained previously for Hopf algebras of rooted trees.

E.K. Narayanan (Indian Institute of Science, Bangalore, India)

Title: A qualitative uncertainty principle on the Heisenberg group

Abstract: Let $\mathbb{H}^n = \mathbb{C}^n \times \mathbb{R}$ be the Heisenberg group. If an integrable function f is supported on a set of the form $B \times \mathbb{R}$ where $B \subset \mathbb{C}^n$ has finite Lebesgue measure and the group Fourier transform $\hat{f}(\lambda)$ is of finite rank for all λ, then f vanishes identically. Extension of this result to two step nilpotent Lie groups will also be discussed. (Joint work with P. K. Ratnakumar)

Yuri A. Neretin (Institute for Theoretical and Experimental Physics, Moscow, Russia)

Title: Multiplications of double cosets for infinite-dimensional groups

Abstract: Let G be a group, K a subgroup. Denote by $K \setminus G/K$ the space of double cosets. If G is compact, then functions on $K \setminus G/K$ form an algebra with respect to a convolution. For special infinite-dimensional groups a set $K \setminus G/K$ quite often is

a semigroup. Moreover, it acts in a natural way in spaces of K-fixed vectors in any unitary representations of G.
Apparently, the first example was a product of operator colligations arising to M.S.Livshits and V.P.Potapov works, 1946-1955. Numerous constructions of representation-theoretic origin were discovered by R.S.Ismagilov, G.I.Olshanski, and speaker.
We describe this sets in some cases. For instance, if $(G, K) = (GL(\infty, R), O(\infty))$ we get a semigroup, whose elements are holomorphic maps from Riemannian sphere to Lagrangian Grassmannian in $\mathbb{C}^{2n} \oplus \mathbb{C}^{2n}$. If $(G, K) = (S(\infty) \times S(\infty), \mathrm{diag}(S(\infty)))$ we get a two-dimensional topological field theory.
We also discuss finite dimensional versions of these operations.

Michael Pevzner (Université de Reims, France)

Title: Geometric analysis on small representations of $GL(N, \mathbb{R})$

Abstract: The most degenerate unitary principal series representations attain the minimum of the Gelfand–Kirillov dimension among all irreducible unitary representations of $GL(N, \mathbb{R})$. We shall discuss irreducible decompositions of their restrictions with respect to all reductive symmetric pairs (G, H). A particular attention will be paid to the case when $H = Sp\,(n, \mathbb{R})$. In this situation we will describe a new model for these representations in which the Knapp-Stein intertwining operators have a simple algebraic interpretation.

Sofiane Souaifi (IRMA, Strasbourg, France)

Title: Paley-Wiener theorem(s) for real reductive Lie groups

Abstract: In the early 80's, J. Arthur proved the Paley-Wiener theorem for real reductive Lie groups. To describe the Fourier transform of the space of compactly supported smooth functions, he uses the so-called Arthur-Campoli relations. More recently, P. Delorme, using other techniques, gave another proof of the Paley-Wiener theorem. His description of the Paley-Wiener space is now in terms of intertwining conditions.
In a joint work with E. P. van den Ban, we make a detailed comparison between the two spaces, without using the proof or any validity of any of the associated Paley-Wiener theorems. This is done by use of the Hecke algebra.

Arnaud Souvay (Université Henri Poincaré, Nancy, France)

Title: From Taylor expansions to Weil functors
(Joint work with Wolfgang Bertram, Nancy)

Abstract: A *Weil algebra* (over a commutative field or ring $\mathbb{K}$) is a finite-dimensional commutative associative algebra of the form $\mathbb{A} = \mathbb{K} \oplus N$, with a niplotent ideal N. Examples are: the *tangent ring* $T\mathbb{K} = \mathbb{K}[X]/(X^2)$, the *jet rings* $J^k\mathbb{K} = \mathbb{K}[X]/(X^{k+1})$, and the *iterated tangent rings* $T^{k+1}\mathbb{K} = T(T^k\mathbb{K})$. These examples of ring extensions of $\mathbb{K}$ correspond to constructions in differential geometry: construction of the tangent bundle TM, of the jet bundle J^kM and of higher order tangent bundles T^kM, respectively.

We will explain that a general Weil algebra $\mathbb{A}$ corresponds to a general *Weil bundle* $T^{\mathbb{A}}M$ of a manifold M, and that this construction has excellent properties: it is not only functorial in M, but also in $\mathbb{A}$, and $T^{\mathbb{A}}M$ is a smooth manifold not only over $\mathbb{K}$, but also over $\mathbb{A}$. Here, the dimension of M over $\mathbb{K}$ and the characteristic of $\mathbb{K}$ may be arbitrary, so that one has to define *jets* and *Taylor polynomials* (which are the main tools for constructing $T^{\mathbb{A}}M$) in a suitable way that does not require division by factorials.

Guillaume Tomasini (Max Planck Institut für Mathematik, Bonn, Germany)

Title: Integrability of degree 1 modules

Abstract: Let $\mathfrak{g}$ be a (semi)simple complex Lie algebra. Let $\mathfrak{h}$ be a Cartan subalgebra. A weight module is a $\mathfrak{g}$-module which is $\mathfrak{h}$-diagonalisable, and whose weight spaces are finite dimensional. A degree 1 module is a weight module whose non trivial weight spaces are 1-dimensional. The aim of this talk is to answer the following question: when does a simple degree 1 module come from a continuous representation (in a Hilbert space) of a real Lie group G whose (complexified) Lie algebra is $\mathfrak{g}$? This is joint work with Bent Orsted.

Lyudmila Turowska (Chalmers University, Göteborg, Sweden)

Title: Beurling-Fourier algebras on compact groups.

Abstract: For a compact group G, I will define the Beurling-Fourier algebras $A_\omega(G)$ on G, for weights $\omega : \hat{G} \to \mathbb{R}^+$. The classical Fourier algebra of G corresponds to the case where ω is the constant weight 1. When G is abelian, $A_\omega(G) = 1_1(\hat{G}; \omega)$, the classical Beurling algebra on $\hat{G}$. To describe the spectrum of $A_\omega(G)$, we require an abstract Lie theory which is built from Krein-Tannaka duality, and was formalized separately by McKennon, and Cartwright and McMullen, in the '70s. This Lie theory allows us to develop the complexification $G_{\mathbb{C}}$, even for non-Lie G. The Gelfand spectrum G_ω can always be realized as a subset of $G_{\mathbb{C}}$. We consider the following questions. When, for a symmetric weight ω, is $A_\omega(G)$ symmetric? When is $A_\omega(G)$ regular on G? Can we gain information on sets of spectral synthesis in G_ω? This is a joint work, with Jean Ludwig and Nico Spronk.

Roger Zierau (Oklahoma State University, USA)

Title: Associated cycles of Harish-Chandra modules

Abstract: An important invariant of a Harish-Chandra module is its assoicated cycle. The assocuated cycle is an algebraic invariant that has an equivalent analytic formulation (in terms of the global character). There is no known method for calculating associated cycles in general, however in this lecture some techniques for the computation of associated cycles of many irreducible Harish-Chandra modules will be discusssed.