

# THE 3rd EUROPEAN GIRLS' MATHEMATICAL OLYMPIAD

## Luxembourg Leader's Report

*Ein luxemburgisches Frühlingsmärchen.*

In mid-April 2014, Turkey hosted the third European Girls' Mathematical Olympiad (or EGMO for initiates), following onto the initial edition of this competition in the United Kingdom in 2012, and the second edition, held in Luxembourg in 2013. Antalya welcomed 110 contestants from 29 countries, among whom were 21 contestants from 6 guest nations from all over the world.

At EGMO 2014, Luxembourg was represented by a team of two contestants, led by Pierre Haas (leader) and Bernard Felten (deputy leader). Charles Leytem, Luxembourg's representative on the EGMO advisory board, also attended the competition. More or, in many cases, less flattering mugshots of participants can be found at the official internet side of EGMO, <http://www.egmo.org>, where the interested reader may find virtually any information on EGMO, ranging from results at past competitions to detailed explanations, cast in soporific legalistic parlance, of what constitutes "force majeure". For those whose intellect craves for something beyond sobre statistics of results, the present report aims at giving a more buoyant description of EGMO.

As its mother competition, the International Mathematical Olympiad (IMO), EGMO is a competition among individuals engaging in the chivalrous fight for shiny metal, but a ranking of nations is published and carefully scrutinised by team leaders. It has been pointed out that Luxembourg has held for many years the position of the leading Grand-Duchy at these competitions. In 2014, Luxembourg ranked 20th among 29 competing countries, winning a bronze medal. The Luxembourg performances were

		P1	P2	P3	P4	P5	P6		
LUX1	Christina Meyer	3	0	0	6	1	0	10	Bronze Medal
LUX2	Tara Trauthwein	3	0	0	2	0	0	5	

For each problem, a maximum mark of seven points was available. This constitutes a very creditable performance for a small country such as Luxembourg: in the unofficial ranking of countries, Luxembourg came above much larger countries such as Belgium, Finland, and Norway, which, unlike Luxembourg, sent a full team of four contestants to the competition.

## A Brief History of EGMO

To dispose of one common misunderstanding straightaway: EGMO is *not* the trade name of a lawn mower that whisks eggs while trimming the lawn; rather, it is the acronym for the European Girls' Mathematical Olympiad. The first question must therefore be: why is there such a thing (a European mathematical competition for girls, mind you, not a household utility of doubtful usefulness)? The reason is simply that, over the past years, about one in ten participants in the International Mathematical Olympiad has been female. This is clearly undesirable<sup>1</sup>; mathematical competitions for girls therefore aim to encourage more girls to take part in mathematical olympiads. One of oldest of these competitions is the Chinese Girls' Mathematical Olympiad (CGMO). EGMO is the brainchild of Geoff Smith, the leader of the United Kingdom at IMO, and its first edition was held in Cambridge (the real one, not the homonymous hamlet in Gloucestershire, and not the one in Massachusetts either) in 2012. The second edition was held in Luxembourg in 2013, followed by third edition in Turkey in 2014 which is the object of the present report. Belarus will host the 2015 edition. While several countries have already expressed their interest at hosting EGMO further in the future, ultimately, EGMO will have been a success when there is no longer any need for it.

The format of EGMO is that of IMO: contestants sit two four-and-a-half-hour papers consisting of three extremely tough mathematical problems on two consecutive days. In a process called coordination, an intense communion between the leaders of the teams and experts from the host country, an integer mark out

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<sup>1</sup>A caveat applies: the Dutch leader at EGMO and chair of the EGMO advisory board, Birgit van Dalen, has related that, when she herself participated in mathematical olympiad training camps, boys got crowded into shared dorms, whereas she, being the only female participant, had a room all to herself.

of seven is assigned to each problem. Medals are awarded to approximately half of the contestants; half of the medals are bronze medals, one third are silver, and one sixth are gold medals. Contestants who do not receive a medal, but solve one question completely, are awarded an honourable mention.

The reason why this competition is a European one is more mundane: sending teams all over the world to participate in a mathematical competition is expensive. Nonetheless, it is heartening to see so many non-European countries wishing to send guest teams to EGMO, and it is hoped that, in this way, EGMO will spawn off many daughter competitions in other parts of the world, just as EGMO itself was modelled partly on CGMO.

## Leader's Diary

*Accepted wisdom holds it that mathematical competitions are best related to others in the form of a leader's diary on the events that may or may not have occurred during the competition. What distinguishes the present diary from its counterparts is the lack of any desire, on the part of the author, to represent his actions in as favourable a light as possible. Caveat lector.*

**Thursday, 10th April 2014.** The EGMO journey began at the still rather new and rather shiny Luxembourg airport, an exceedingly large construction completed just before the words “economic crisis” started to dominate state budgets all over the world, and which replaced the previous airport, a construction dating from the seventies and of limited aesthetic appeal that had elicited slighting comparisons to the airports of various third-world countries from travel magazines. The first leg of our journey to Antalya took us to Istanbul. Turkish Airlines has excellent catering: I had my first taste of *bulgur* (a kind of cereal made from wheat groats) of which I was to consume ample quantities over the coming days. At Istanbul airport, as we were heading towards the (international) passport control, we saw the Finnish (easily recognisable by their immaculately clad deputy leader) and Polish teams who were hurrying in the opposite direction to get to the domestic passport control which had smaller queues. We shared our flight to Antalya with part of the Irish team and the US deputy leader. Clearly, we were heading into the right direction.

Once arrived in Antalya, we were shepherded into a bus that took us to the base of EGMO 2014: a five-star beach resort in the town of Belek, about half an hour's drive away from Antalya. Compared with the Luxembourg youth hostel in which EGMO 2013 was based, these surroundings were rather grand. We also met our team guide, Hatice, born in Germany to Turkish immigrants who had later returned to Turkey. She spoke very good German and proved to be an excellent guide to the team.

**Friday, 11th April 2014.** While the deputy leaders and contestants were left to enjoy a prolonged breakfast, the day started off for the leaders with the first jury meeting, at which the problems selected by the Problem Selection Committee were approved<sup>2</sup>:

### Day One

**Problem 1.** Determine all real constants  $t$  such that whenever  $a, b, c$  are the lengths of the sides of a triangle, then so are  $a^2 + bct$ ,  $b^2 + cat$ ,  $c^2 + abt$ .

**Problem 2.** Let  $D$  and  $E$  be points in the interiors of sides  $AB$  and  $AC$ , respectively, of a triangle  $ABC$ , such that  $DB = BC = CE$ . Let the lines  $CD$  and  $BE$  meet at  $F$ . Prove that the incentre  $I$  of triangle  $ABC$ , the orthocentre  $H$  of triangle  $DEF$  and the midpoint  $M$  of the arc  $BAC$  of the circumcircle of triangle  $ABC$  are collinear.

**Problem 3.** We denote the number of positive divisors of a positive integer  $m$  by  $d(m)$  and the number of distinct prime divisors of  $m$  by  $\omega(m)$ . Let  $k$  be a

<sup>2</sup>Unlike IMO, where the six problems that make up the contest papers are selected from a shortlist of thirtyish problems by the team leaders, at EGMO, the Problem Selection committee actually sets the papers (with a list of backup problems in case any problem gets rejected on the ground of being already known). While this has its advantages (for instance, at IMO, the team leaders do not have that much time to select the problems so that the final selection might not be that well balanced), the Problem Selection Committee might blunder, which, as expatiated upon later in the report, happened at this EGMO, and set papers that are far too hard. In the future, some external control of the papers before the first jury meeting (e.g. by the EGMO advisory board) might therefore be desirable.

positive integer. Prove that there exist infinitely many positive integers  $n$  such that  $\omega(n) = k$  and  $d(n)$  does not divide  $d(a^2 + b^2)$  for any positive integers  $a, b$  satisfying  $a + b = n$ .

### *Day Two*

**Problem 4.** Determine all integers  $n \geq 2$  for which there exist integers  $x_1, x_2, \dots, x_{n-1}$  satisfying the condition that if  $0 < i, j < n$ ,  $i \neq j$  and  $n$  divides  $2i + j$ , then  $x_i < x_j$ .

**Problem 5.** Let  $n$  be a positive integer. We have  $n$  boxes where each box contains a non-negative number of pebbles. In each move we are allowed to take two pebbles from a box we choose, throw away one of the pebbles and put the other pebble in another box we choose. An initial configuration of pebbles is called *solvable* if it is possible to reach a configuration with no empty box, in a finite (possibly zero) number of moves. Determine all initial configurations of pebbles which are not solvable, but become solvable when an additional pebble is added to a box, no matter which box is chosen.

**Problem 6.** Determine all functions  $f: \mathbb{R} \rightarrow \mathbb{R}$  satisfying the condition

$$f(y^2 + 2xf(y) + f(x)^2) = (y + f(x))(x + f(y))$$

for all real numbers  $x$  and  $y$ .

These problems had been proposed by the United Kingdom, Ukraine, Japan, the Netherlands, Romania, and the Netherlands, respectively. After this feat, the jury clapped (because everyone had managed to raise their voting sticks at the correct moment), and then adjourned to attend the opening ceremony. The speeches were mercifully short, and a female presenter, blatantly provided by the headquarters of TUBITAK (the Turkish scientific research council that organised the olympiad and did a fantastic job of it), beamed at the audience in the intervals between speeches, showing her immaculate teeth and the kind of smile that would sprain an ordinary person's jaw muscles.

In the afternoon, the jury set about to translate the contest papers into their various languages<sup>3</sup> and to have a go at the problems themselves. Although I solved problem 2 rather quickly by spotting a pair of homothetic parallelograms, I found the supposedly easy questions 1 and 4 much more unyielding. As the afternoon progressed, it gradually transpired that other leaders were struggling, too, and at dinner-time, it had become clear that the papers were really quite hard. The unintermittent downpour that later turned into a thunderstorm imparted a suitably *Götterdämmerung*-like feel to the atmosphere in the jury room. At dinner, the contestants were still quite cheerful, blissfully unaware of the doom that lay in waiting for them.

**Saturday, 12th April 2014.** With the first day of the contest came much improved weather conditions, and a post-breakfast stroll along the beach offered glorious views of snow-covered mountain tops on the horizon. After the questions-and-answers session (no question from the Luxembourg contestants), Geoff Smith informed me that he thought that the jury was being rather quiet (presumably referring to the fact that none of his admittedly unfailingly sensible comments starting with "If the jury allows me to speak" had been shouted down), and that something had to be done about that. Accordingly, I spoke against the proposed markscheme for the first question until the coordinates, wearied by the sheer quantity of my own and other people's expostulations, agreed to change the markscheme to something I deemed more sensible. Charles Leytem, now wearing the mantle of the problem captain of the second problem, adopted the much more successful approach of talking the jury in painstaking detail through a very comprehensive markscheme for that question, thereby sending the jury to sleep. The markscheme for third question was similarly waved through, as the jury was mindful of the approaching time for lunch.

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<sup>3</sup>I took the easy way out and let the French and Belgian leaders prepare the French version. However, sometimes, the easy way out is not available: at EGMO 2012, I was the only German-speaking leader (for the Swiss leader only spoke French), so I had to prepare the German version of the contest papers for the Swiss contestants, a rather unwelcome task, because, although I speak fluent German, I have never done mathematics in German.

Later in the afternoon, having allowed time for our digestive systems to deal with the vaguely unhealthy amounts of Turkish sweets that we were ingesting, Charles Leytem and I went to the swimming pool, where we were joined soon by the Belgian contestants, whose arrival was heralded by a Dirac-delta-like peak in the sound intensity landscape. They greeted the slightly later arrival of Pierre-Alain Jacqmin, the Belgian deputy leader, with shouts of “*Elle est bonne, elle est bonne!*”. Having spent some time in the pool by then, I had come to the conclusion that the water temperature was everything but “*bonne*” (and at the time of writing these lines, I am still nursing a rather persistent bout of bronchitis). Pierre-Alain must have come to a similar conclusion about the water temperature, for, when the Belgian leader arrived at the poolside, he started to shout “*Elle est bonne, elle est bonne!*”, too, with a rather big grin on his face. After some paddling in the pool, in search of greater challenges, we decided to brave the waterslides, where we met up with the Luxembourg contestants. Later, the assembled Luxembourg and Belgian leaderships were cornered by the cameramen of EGMO who were somehow endowed with the gift of ubiquity. Fortunately, since the darts competition was about to start, the Belgian contestants increased their sound production once more, upon which the camera team decided that they were rather more photogenic than us. When the first darts, having flown in rather alarmingly random directions, started to miss the dartboard, I returned to the jury room to indulge in a bit of marking.

After dinner, I joined the Luxembourg contestants on a walk along the beach. To boost morale, we sang uplifting songs such as “Empty chairs at empty tables” from *Les Misérables* (though I thought that my falsetto rendition of “There is a castle on a cloud”, a desperate attempt to match LUX1’s trained soprano voice, was rather something to write home about).

**Sunday, 13th April 2014.** Having dealt with the contestants’ questions (one question from LUX1, a detail that was only to unveil its importance later on) and having approved the markschemes, the leaders and deputy leaders and other dignitaries of EGMO were carted off on an excursion, while the contestants were still sweating (figuratively, I guess, because of the air conditioning in the exam room) over the second paper. After lunch, during which Geoff Smith explained the subtleties of mathematical olympiad terminology (“observer with leader” really means “observer of leader”), we visited a waterfall. Our guide, suffering from that kind of acute verbal incontinence that mathematicians generally respond badly to, led us via a rather slippery path down to the bottom of the waterfall, so that we could have a better, if somewhat wetter view. When a large drop of water made contact with an exposed part of my neck, I cried out (rather, and, with hindsight, disproportionately) loudly, thereby scaring the living daylights out of Jo Harbour, the UK deputy. Charles Leytem and I decided to investigate whether we could get a better view of the top of the waterfall by climbing a rather steep and overgrown path on the side and crossing a dilapidated wooden bridge. This turned out not to be the case.

The last stage of our excursion was a visit to the old city centre of Antalya, tucked away rather picturesquely in a natural marina that had been the harbour of Antalya during the Roman era, and now harbouring a plethora of shops offering everything from spices to underwear bearing the names of German professional football teams (which left us in no doubt as to where the vast majority of tourists visiting the area hails from). In an extraordinary display of lack of foresight, Antalya’s city fathers had had the city walls of Antalya pulled down in the early twentieth century. One of the few remnants of the days of yore is Hadrian’s gate, which we eventually managed to locate after a rather circuitous wander through the city. We should have tried to get a map of the city centre after all.

We spent the evening marking, LUX1 having done James Joyce proud. When we finally decided that we had squeezed all marks out of our contestants’ scripts, it was past midnight. At this time, other leaders were still going strong. The Dutch leadership, who had to mark the attempts of the two Turkish teams at the two Dutch questions on the second day’s paper, certainly had a fun few hours ahead of them.

**Monday, 14th April 2014.** While the leaders and deputy leaders braced themselves to face the coordinators, the contestants were carted off on an excursion (they did not visit Antalya city centre, but were taken to the ruins of Perge instead). Coordination at EGMO is generally a relaxed business. Unfortunately, there was the matter of coordinating Question 4, where we claimed a full solution from LUX1. After a lengthy discussion, during which one of the coordinators repeatedly asked for translations of very random bits of her

work, we were informed that they were offering five marks. I objected, remarking that LUX1 was at most guilty of a sin of omission (she had said “We are done.” rather than “By an obvious result from graph theory, we are done.”), but certainly not of a sin of commission, yet the coordinators would not budge. The whole matter was somewhat unfortunate for it was somewhat linked up with the answer that the jury had agreed to send to LUX1’s query during the contest. Given the lack of sleep the previous night, I was having none of that. Forgetting all about Geoff Smith’s pleas for a non-confrontational coordination process, I informed that coordinators that we were getting nowhere, jumped up and stormed out of the room, leaving a rather dazzled Bernard Felten to scoop up the various bits of paper that I had left on the coordination table. (As the latter later put it, this was the first time he had ever seen me lose my *sang-froid*.) Well, we garnered support from other leaders, and, returning to the coordination table later in the afternoon, with the Damocles sword of the coordinators making a fool of themselves if this matter were referred to the jury hanging portentously over the coordination table like a precariously fastened lustre, got the coordinators to agree to six marks for LUX1. I feel that this was fair.

The final jury meeting (to formally approve the results of the competition and settle the matter of the medal boundaries) took place in the evening. After looking through the results, I had come to the conclusion that the appropriate bronze cut-off was eight marks, for this was the highest cut-off that would still ensure that everyone who had essentially solved one question would get a medal. I inadvertently wrought a certain amount of havoc by starting a discussion on the medal boundaries just as the organisers were about to put forth their own suggestions (with which I happened to disagree). Anyway, the lower boundaries were voted through (probably as a consequence of a general tendency to allow for lower medal boundaries if the contest was too hard).

**Tuesday, 15th April 2014.** After we had spent some more time at the pool (which, incidentally, had not warmed up since our last visit), it was time for the closing ceremony. The speeches were appropriately short, the medals were awarded, and the Olympiad formally ended with the handing over of the EGMO flag to next year’s organisers, Belarus, who also showed, as a kind of appetiser, a promotional video blatantly provided, together with their deputy leader, by the Belorussian Ministry of Education.

After the ceremony, we were taken by bus to a harbour, where we boarded a ship for the farewell party. Somewhat disappointingly, we did not board the *faux*-pirate ship anchored next to our ship (our ship did however have a parrot on board). During the pre-prandial drinks on the top deck, the US leadership did some gymnastics on a metal contraption until they were told off by the captain, and the UK deputy leader decided that, in order to widen her cultural horizons, she should talk to some leaders who were not native speakers of English. She settled on the only leader who lives in the same town in the UK as her.

Dinner was served on the lower deck, as we descended, we had to pass the nightclub-themed middle deck, from which rather impressively-sized loudspeakers were blaring the kind of noise some (and often young) people consider to be music at the unsuspecting maritime wildlife. This was boding well. It turned out that musical purgatory continued on the lower deck, in the form of a band delivering classics such as Mary Hopkin’s “Once upon a time there was a tavern” at the top of their voice. My scheming to put an end to this acoustic assault by pulling at some strategically chosen power cables was cut short when Geoff Smith went up to the band and voiced his disapproval while frantically waving. It did not do much good; presumably, they thought that he was dancing. Later, we did some actual dancing, and, as the music on the middle deck grew more and more cheesy, the latter became an acceptable place to be, too.

As we were heading back into the harbour, and since the hour had grown late, our discussion turned to more philosophical topics, such as the concept of power in Tolkien’s legendarium. Geoff Smith pointed out that Tolkien had overlooked the devastating effects of the power of a point.

**Wednesday, 16th April 2014.** By applying a suitable amount of pressure (equal to my body weight), I managed to fit all my belongings, which, by then, had grown to include the various goodies and the scripts of the Luxembourg contestants, into my suitcase. By then, it was past one o’clock in the morning, so I went down to the hotel lobby to await the arrival of the bus that was to take us to airport shortly after the ungodly hour of three o’clock in the morning. Charles Leytem and Bernard Felten slept the sleep of the innocent, while the Luxembourg contestants were chatting to an international assortment of other contestants. I

stayed awake in the company of a cup of coffee, a cup of hot chocolate, and some chapters of Hilary Mantel's *magnum opus* on the French revolution, "A Place of Greater Safety". Of these three items, the latter two were soporifics.

Of the journey home, I remember but short moments of wakefulness, and the in-flight movie "Saving Mr. Banks" on the flight from Istanbul to Luxembourg. At the airport, I handed the scripts back to the team. Rather surprisingly, they had survived the pressure that I had applied (mind you, I am talking of the folders containing the scripts and the pressure required to close by suitcase). In fact, a bottle of liquid soap in my suitcase had not.

I should like to thank those that have not only made EGMO possible, but also made it an extremely enjoyable event. Were I to try and list these people, I know I would miss a few key players. Therefore, I am only going to thank the Luxembourg contestants, for being very pleasant company, and for defending our national colours so valiantly.

Cambridge, 19th May 2014.