ON THE RELATIONSHIP OF MAXIMAL CLONES AND MAXIMAL C-CLONES

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C-clones are clones that can be described as polymorphism sets of relations of the form $\mathrm{R}_{\mathbf{b}}^{\mathbf{a}} := \{\,(\mathbf{x},\mathbf{y}) \in D^{p+q} \mid \exists 1 \leq i \leq p\colon x_i \geq a_i \vee \exists 1 \leq j \leq q\colon y_j \leq b_j\}$, where the tuples $\mathbf{a} \in D^p$, $\mathbf{b} \in D^q$ are parameters for $p, q \in \mathbb{N}_+$, and where $\leq$ denotes the canonical linear order on the finite set $D = \{0, \ldots, n-1\}$. As clones in general, also C-clones form a complete lattice w.r.t. set inclusion, and the co-atoms of this lattice are called *maximal C-clones*.

On a Boolean domain it is known that there exist exactly five C-clones, and precisely one of them is maximal: this unique maximal C-clone coincides with the clone of monotone Boolean functions. However, it has been shown that on larger domains this statement ceases to hold: not only are there infinitely many C-clones whenever $|D| \geq 3$, but also is none of the maximal clones a C-clone. Hence, in this case every maximal C-clone is a proper subset of some maximal clone.

In this talk, we will study the exact relationship between maximal C-clones and maximal clones with regard to set inclusion. It turns out that some types of maximal clones do not contain any maximal C-clone and thus no C-clone at all. Surprisingly, we can show that for each maximal C-clone there exists a unique maximal clone in which it is contained. As a consequence, we can also derive a new completeness criterion for C-clones.

This talk presents results that have been jointly obtained with Edith Vargas-García, Universidad Autónoma de la Ciudad de México and University of Leeds.