ON THE SEMISIMPLICITY OF GEOMETRIC MONODROMY ACTION IN $\mathbb{F}_\ell$-COEFFICIENTS

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Abstract: Let $X/\mathbb{F}_q$ be a smooth separated geometrically connected variety with $\bar{x}$ a geometric point, $f : Y \to X$ a smooth projective morphism, and $w \geq 0$ an integer. We compare the invariant dimensions of sufficiently many ℓ-adic and mod ℓ representations of the geometric étale fundamental group $\bar{\Pi} := \pi_1^{\text{ét}}(X_{\bar{\mathbb{F}}_q}, \bar{x})$ arising from the higher direct image sheaves $R^w f_* \mathbb{Q}_\ell$ and $R^w f_* \mathbb{F}_\ell$ respectively. These dimension data implies that the mod ℓ representation

$$\rho_\ell : \bar{\Pi} \to \mathrm{GL}(H^w(Y_{\bar{x}}, \mathbb{F}_\ell))$$

is semisimple for $\ell \gg 0$. This is a joint work with Anna Cadoret and Akio Tamagawa.