B Computing Hecke algebras of weight 1 in MAGMA

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B.1 Introduction

The aim of this appendix is twofold. On the one hand, we report on an implementation in MAGMA (see [4]) of a module for the Hecke algebra of Katz cusp forms of weight 1 over finite fields, which is based on section 4 of this article.

On the other hand, we present results of computations done in relation with the calculations performed by Mestre (see appendix A) in 1987.

The program consists of two packages, called *Hecke1* and *CommMatAlg*. The source files and accompanying documentation ([42] and [43]) can be downloaded from the author's homepage (`http://www.math.leidenuniv.nl/~gabor/`).

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B.2 Algorithm

In the current release MAGMA ([4]) provides William Stein's package HECKE, which contains functions for the computation of Hecke algebras and modular forms over fields. There is, however, the conceptual restriction to *weights greater equal* 2.

Edixhoven's approach for the construction of a good weight 1 Hecke module, which is at the base of the implemented algorithm, relates the Hecke algebra of characteristic p Katz cusp forms of weight 1 to the Hecke algebra of classical weight p cusp forms over the complex numbers. The latter can for instance be obtained using modular symbols.

Katz modular forms

Following the notations of section 4, we denote by $S_k(\Gamma_1(N), \overline{\epsilon}, \mathbb{F})'_{\mathrm{Katz}}$ the space of *Katz cusp forms* of weight k, level N, with character $\overline{\epsilon} : (\mathbb{Z}/N)^* \to \mathbb{F}^*$ over the $\mathbb{Z}[1/N]$-algebra R, where we impose that $k \geq 1$ and $N \geq 5$. For a definition see section 4 or [27] for more details.

By the space of *classical cusp forms* $S_k(\Gamma_1(N), \epsilon, R)$ over a ring $R \subseteq \mathbb{C}$, we understand the sub-R-module of $S_k(\Gamma_1(N), \epsilon, \mathbb{C})$ consisting of the forms with Fourier coefficients (at infinity) in the ring R.

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Let us mention that for a homomorphism of $\mathbb{Z}[1/N]$-algebras $R \to S$, we have the isomorphism ([27], Prop. 2.5, and the proof of [17], Thm. 12.3.2)

$$S_k(\Gamma_1(N), R)'_{\mathrm{Katz}} \otimes_R S \cong S_k(\Gamma_1(N), S)'_{\mathrm{Katz}},$$

if $k \geq 2$ or if $R \to S$ is flat. Using the statements in 4.7, it follows in particular that we have the equality

$$S_k(\Gamma_1(N), R)'_{\mathrm{Katz}} = S_k(\Gamma_1(N), R),$$

in case that $\mathbb{Z}[1/N] \subseteq R \subseteq \mathbb{C}$ or $k \geq 2$.

Modular symbols

Given integers $k \geq 2$ and $N \geq 1$, one can define the complex vector space $\mathcal{S}_k(\Gamma_1(N))$ of *cuspidal modular symbols* (see e.g. [36], section 1.4). On it one has in a natural manner Hecke and diamond operators, and there is a non-degenerate pairing

$$\left(S_k(\Gamma_1(N), \mathbb{C}) \oplus \overline{S_k(\Gamma_1(N), \mathbb{C})}\right) \times \mathcal{S}_k(\Gamma_1(N)) \to \mathbb{C},$$

with respect to which the diamond and Hecke operators are adjoint (see [36], Thm. 3 and Prop. 10).

We recall that the diamond operators provide a group action of $(\mathbb{Z}/N)^*$ on the above spaces. For a character $\epsilon : (\mathbb{Z}/N)^* \to \mathbb{C}^*$ one lets, in analogy to the modular forms case, $\mathcal{S}_k(\Gamma_1(n), \epsilon)$ be the ϵ-eigenspace.

Let $\mathbb{Z}[\epsilon]$ be the smallest subring of $\mathbb{C}$ containing all values of ϵ. It follows that the $\mathbb{Z}[\epsilon]$-algebra generated by all Hecke operators acting on $S_k(\Gamma_1(N), \epsilon, \mathbb{C})$ is isomorphic to the one generated by the Hecke action on $\mathcal{S}_k(\Gamma_1(n), \epsilon)$. The same applies to the $\mathbb{Z}$-algebra generated by the Hecke operators on the full spaces (i.e. without a character).

Notation B.2.1 We call the Hecke algebras described here above $\mathbb{T}(\epsilon)$ and $\mathbb{T}$ respectively.

It is known (for the method see e.g. Prop. 4.2) that the first Bk Hecke operators suffice to generate $\mathbb{T}(\epsilon)$, where the number B is $\frac{N}{12}\prod_{l|N, l \text{ prime}}(1+\frac{1}{l})$. For the full Hecke algebra $\mathbb{T}$ one has to take $Bk\varphi(N)/2$.

Weight 1 as subspace in weight p

Let us assume the following

Setting B.2.2 Let K be a number field, $\mathcal{O}_K$ its ring of integers, $\mathfrak{P}$ a prime of $\mathcal{O}_K$ above the rational prime p and $N \geq 5$ an integer coprime to p. Moreover, we consider a character $\epsilon : (\mathbb{Z}/N)^* \to \mathcal{O}_K^*$. For a given field extension $\mathbb{F}$ of $\mathcal{O}_K/\mathfrak{P}$, we fix the canonical ring homomorphism $\phi : \mathcal{O}_K \twoheadrightarrow \mathcal{O}_K/\mathfrak{P} \hookrightarrow \mathbb{F}$. We denote by $\overline{\epsilon}$ the composition of ϵ with ϕ. Recall that B was defined to be $\frac{N}{12}\prod_{l|N, l \text{ prime}}(1+\frac{1}{l})$.

We shall quickly explain how Edixhoven relates weight 1 to weight p in section 4 in order to be able to formulate our statements.

The main tool is the *Frobenius* homomorphism $F : S_1(\Gamma_1(N), \mathbb{F}_p)'_{\mathrm{Katz}} \to S_p(\Gamma_1(N), \mathbb{F}_p)'_{\mathrm{Katz}}$ defined by raising to the p-th power. Hence on q-expansions it acts as $a_n(Ff) = a_{n/p}(f)$, where $a_{n/p}(f) = 0$ if $p \nmid n$. Also by F we shall denote the homomorphism obtained by base extension to $\mathbb{F}$. One checks that F is compatible with the character. The sequence of $\mathbb{F}$-vector spaces

$$0 \to S_1(\Gamma_1(N), \overline{\epsilon}, \mathbb{F})'_{\mathrm{Katz}} \xrightarrow{F} S_p(\Gamma_1(N), \overline{\epsilon}, \mathbb{F})'_{\mathrm{Katz}} \xrightarrow{\Theta} S_{p+2}(\Gamma_1(N), \overline{\epsilon}, \mathbb{F})'_{\mathrm{Katz}}$$

is exact, where Θ denotes the derivation described before Prop. 4.2. The image of F in $S_p(\Gamma_1(N), \overline{\epsilon}, \mathbb{F})'_{\mathrm{Katz}}$ is effectively described by Prop. 4.2 to be those $f \in S_p(\Gamma_1(N), \overline{\epsilon}, \mathbb{F})'_{\mathrm{Katz}}$ such that $a_n(f) = 0$ for all n with $p \nmid n$, where it suffices to take $n \leq B(p+2)$ with B as before.

Using the homorphisms

$$\begin{aligned} S_1(\Gamma_1(N), \overline{\epsilon}, \mathbb{F})'_{\mathrm{Katz}} \overset{F}{\hookrightarrow} S_p(\Gamma_1(N), \overline{\epsilon}, \mathbb{F})'_{\mathrm{Katz}} &\overset{4.6}{\cong} \big((S_p(\Gamma_1(N), \mathbb{Z})) \otimes_{\mathbb{Z}} \mathbb{F}\big)(\overline{\epsilon}) \\ &\overset{4.8}{\cong} \big(\mathrm{Hom}_{\mathbb{Z}}(\mathbb{T}, \mathbb{Z}) \otimes_{\mathbb{Z}} \mathbb{F}\big)(\overline{\epsilon}) \cong (\mathbb{T} \otimes_{\mathbb{Z}} \mathbb{F})^\vee(\overline{\epsilon}), \end{aligned}$$

one obtains an isomorphism of Hecke modules (cp. Thm. 4.9)

$$S_1(\Gamma_1(N), \overline{\epsilon}, \mathbb{F})'_{\mathrm{Katz}} \cong \big((\mathbb{T} \otimes_{\mathbb{Z}} \mathbb{F})/\tilde{\mathcal{R}}\big)^\vee, \tag{B.2.1}$$

where $\tilde{\mathcal{R}}$ denotes the sub-$\mathbb{F}$-vector space of $\mathbb{T} \otimes_{\mathbb{Z}} \mathbb{F}$ generated by $1 \otimes \overline{\epsilon}(l) - < l > \otimes 1$ for $(l, N) = 1$ and by T_n for $n \leq B(p+2)$ and $p \nmid n$. The action of the Hecke operators is the same as the one given in the proposition below.

We would like to replace the full Hecke algebra $\mathbb{T}$, which is expensive to calculate, by $\mathbb{T}(\epsilon)$. One has a natural surjection $\mathbb{T} \otimes_{\mathbb{Z}} \mathbb{Z}[\epsilon] \twoheadrightarrow \mathbb{T}(\epsilon)$, which sends $< l > \otimes 1$ to $\epsilon(l) \cdot \mathrm{id}$.

Proposition B.2.3 Assume the setting B.2.2 and the notation B.2.1. Let $\mathcal{R}$ be the sub-$\mathbb{F}$-vector space of $\mathbb{T}(\epsilon) \otimes_{\mathbb{Z}[\epsilon]} \mathbb{F}$ generated by $T_n \otimes 1$ for those $n \leq B(p+2)$ not divisible by p. Then there is an injection of Hecke modules

$$\big((\mathbb{T}(\epsilon) \otimes_{\mathbb{Z}[\epsilon]} \mathbb{F})/\mathcal{R}\big)^\vee \hookrightarrow S_1(\Gamma_1(N), \overline{\epsilon}, \mathbb{F})'_{\mathrm{Katz}}.$$

For a prime $l \neq p$, the natural action of the Hecke operator T_l in weight p corresponds to the action of T_l in weight 1. The natural action of the operator $T_p + \overline{\epsilon}(p)F$ on the left corresponds to the action of T_p in weight 1. Here $F : \mathbb{T}(\epsilon) \otimes_{\mathbb{Z}[\epsilon]} \mathbb{F} \to \mathbb{T}(\epsilon) \otimes_{\mathbb{Z}[\epsilon]} \mathbb{F}$ sends $T_n \otimes 1$ to $T_{n/p} \otimes 1$ with the convention $T_{n/p} \otimes 1 = 0$ if p does not divide n.

Proof. With $\mathbb{T}$ and $\tilde{\mathcal{R}}$ as defined before the proposition, we have a surjection

$$(\mathbb{T} \otimes_{\mathbb{Z}} \mathbb{F})/\tilde{\mathcal{R}} \twoheadrightarrow (\mathbb{T}(\epsilon) \otimes_{\mathbb{Z}[\epsilon]} \mathbb{F})/\mathcal{R}.$$

Now taking $\mathbb{F}$-vector space duals together with equation B.2.1 gives the claimed injection. The explicit form of the operators follows immediately from equation 4.1.2. $\square$

We treat a special case separately.

Corollary B.2.4 Take in propostion B.2.3 the trivial character 1 and $p = 2$. Then the injection is an isomorphism if there is a prime q dividing N such that $q \equiv 3$ modulo 4.

Proof. As in the proof of Thm. 5.6, one shows that the Hecke algebra of $S_2(\Gamma_0(N), \mathbb{F}_2)'_{\text{Katz}}$ is $\mathbb{T}(1) \otimes \mathbb{F}_2$. Hence, we have

$$\big(\mathbb{T}(1) \otimes \mathbb{F}_2\big)^\vee \cong \big(\mathbb{T} \otimes \mathbb{F}_2/(1 \otimes 1 - < l > \otimes 1 \mid (l, N) = 1)\big)^\vee,$$

whence the corollary follows. $\square$

B.3 Software

Functionality

In this section we wish to present, in a special case, what *Hecke1* computes. Please consult section 2 of [42] for precise statements.

INPUT: Let C be the space of cuspidal modular symbols of weight $p = 2$ and odd level $N \geq 5$ for the trivial character over the rational numbers.

COMPUTE: Let $\phi : \mathbb{Z} \to \mathbb{F}_2$ be the canonical ring homomorphism. We denote by $\overline{T_i}$ the image under ϕ of the matrix representing the i-th Hecke operator T_i acting on the natural integral structure of C. Define $\mathrm{Bound} = \frac{1}{3} N \prod_{l|N, l \text{ prime}}(1 + \frac{1}{l})$, so that the subgroup of $\mathbb{Z}^{D \times D}$ (for D the dimension of C) generated by matrices representing the T_n for $n \leq \mathrm{Bound}$ equals the Hecke algebra of weight 2. Let $\mathcal{A}$ be the sub-$\mathbb{F}_2$-vector space of $\mathbb{F}_2^{D \times D}$ generated by $\overline{T_n}$ for $n \leq \mathrm{Bound}$. Define $\mathcal{R}$ to be the subspace of $\mathcal{A}$ generated by $\overline{T_n}$ for all odd $n \leq \mathrm{Bound}$.

Using the natural surjection $< T_i \mid i \leq \mathrm{Bound} > \otimes_{\mathbb{Z}} \mathbb{F}_2 \twoheadrightarrow \mathcal{A}$, it follows immediately from the results of the preceding section that the $\mathbb{F}_2$-vector space

$$\mathcal{H} = \mathcal{A}/\mathcal{R}$$

is equipped with an action by the Hecke algebra of $S_1(\Gamma_0(N), \mathbb{F}_2)'_{\mathrm{Katz}}$ similar to the one explained in proposition B.2.3.

The function `HeckeAlgebraWt1` of *Hecke1* computes this module $\mathcal{H}$ and also the first Bound Hecke operators of weight 1 acting on it. More precisely, a record containing the necessary data is created. Properties can be accessed using e.g. the commands `Dimension`, `Field`, `HeckeOperatorWt1`, `HeckeAlgebra` and `HeckePropsToString`. Please consult [42] (and [43]) for a precise documentation of the provided functions.

An example session

We assume that the packages *CommMatAlg* and *Hecke1* are stored in the folder `PATH`. We attach the packages by typing

```
> Attach ("PATH/CommMatAlg.mg");
> Attach ("PATH/Hecke1.mg");
```

We can now create a record containing all information for computations of Hecke operators of weight 1 acting on $\mathcal{H}$ (as described above with $N = 491$ and $p = 2$).

```
> M := ModularSymbols (491,2);
> h := HeckeAlgebraWt1 (M);
```

It is not advisable to access information by printing `h`. Instead, we proceed as follows:

```
> Dimension(h);
6
> Bound(h);
164
```

These functions have the obvious meanings. If one is interested in some properties of the Hecke algebra acting on $\mathcal{H}$, one can use:

```
> HeckePropsToString(h);
Level N = 491:
***************************

Dimension = 6
```

```
Bound = 164
Class number of quadratic extension with |disc| = 491 is:  9
There are 2 local factors.

Looking at 1st local factor:

Residue field = GF(8)
Local dimension = 3
UPO = 1
Eigenvalues = { 1, w, w^2, w^4, 0 }
Number of max.  ideals over residue field = 3

Looking at 2nd local factor:

Residue field = GF(2)
Local dimension = 3
UPO = 3
Eigenvalues = { 0, 1 }
Number of max.  ideals over residue field = 1
```

Here w stands for a generator of the residue field in question. For the significance of these data, please see the following section.

B.4 Mestre's calculations

In this section we report on computations we performed in relation with Mestre's calculations exposed in appendix A. Mestre considered weight 1 modular forms for $\Gamma_0(N)$, where N is an odd prime.

According to the modified version of Serre's conjecture (see e.g. [22]), one expects that for any 2-dimensional irreducible Galois representation

$$\rho : G_{\mathbb{Q}} \to \mathrm{SL}_2(\overline{\mathbb{F}_2}),$$

which is *unramified at* 2, there exists a *weight* 1 Hecke eigenform $f \in S_1(\Gamma_0(N_\rho), \overline{\mathbb{F}_2})'_{\mathrm{Katz}}$ giving rise to the representation ρ via Deligne's theorem. Here N_ρ is the Artin conductor of the representation ρ.

Unfortunately, the implication ρ is modular, hence ρ comes from a form of weight 1 and level N_ρ is unproved in the exceptional case $p = 2$.

There is a simple way to produce Galois representations, which are unramified at 2, with given Artin conductor N, when N is odd and square-free. One considers the quadratic field $K = \mathbb{Q}(\sqrt{N})$ resp. $K = \mathbb{Q}(\sqrt{-N})$ if $N \equiv 1\ (4)$ resp. $N \equiv 3\ (4)$, which has discriminant (N). Let now L be the maximal subfield of the Hilbert class field of K such that $[L : K]$ is odd. Then L is Galois over $\mathbb{Q}$ of degree $2u$ with u the odd part of the class number of K. The Galois groups in question form a split exact sequence $0 \to G_{L|K} \to G_{L|\mathbb{Q}} \to G_{K|\mathbb{Q}} \to 0$. The conjugation action of $G_{K|\mathbb{Q}}$ via the split on $G_{L|K}$ is by inversion. For any character $\chi : G_{L|K} \to \overline{\mathbb{F}_2}^*$, one has the induced representation $\mathrm{Ind}_{G_{L|K}}^{G_{L|\mathbb{Q}}}(\chi) : G_{L|\mathbb{Q}} \to \mathrm{SL}_2(\overline{\mathbb{F}_2})$. It is irreducible if χ is non-trivial, and $\mathrm{Ind}_{G_{L|K}}^{G_{L|\mathbb{Q}}}(\chi_1) \cong \mathrm{Ind}_{G_{L|K}}^{G_{L|\mathbb{Q}}}(\chi_2)$ if and only if $\chi_1 = \chi_2$ or $\chi_1 = \chi_2^{-1}$. The Artin conductor of $\mathrm{Ind}_{G_{L|K}}^{G_{L|\mathbb{Q}}}(\chi)$ is N. Consequently, one receives $(u-1)/2$ non-isomorphic Galois representations with dihedral image and Artin conductor N. More precisely, the image of $\mathrm{Ind}_{G_{L|K}}^{G_{L|\mathbb{Q}}}(\chi)$ is $D_{2\cdot \#\,\mathrm{Image}(\chi)}$. These are the dihedral representations to which Mestre refers in appendix A.

It is known that any dihedral representation $\mathrm{Ind}_{G_K}^{G_\mathbb{Q}}(\chi)$ is modular, where $K|\mathbb{Q}$ is a quadratic field and $\chi : G_K \to \overline{\mathbb{F}_2}^*$ is a character. However, as mentioned above, the weight and the level are not known to occur as predicted. Looking at the standard proof (see e.g. [12], Theorem 3.14) of modularity, we see that obstacles occur if K is real and does not allow any non-real unramified quadratic extension.

A feature of modular forms over fields of positive characteristic is that even for prime levels the Hecke algebra can be non-reduced. The Hecke algebra is finite-dimensional and commutative, hence it splits into a direct product $\mathbb{T} = \prod_{i=1}^{r} \mathbb{T}_i$ of local algebras. For a local algebra $\mathbb{T}_i$ with maximal ideal $\mathfrak{m}_i$, we introduce the number $\omega(\mathbb{T}_i) = \min\{\ n \mid (\mathfrak{m}_i)^n = (0)\ \}$. It is related to the number $B(m)$ considered in appendix A: one is in the case $B(m)$ with $m \leq \omega(\mathbb{T}) := \max_i(\omega(\mathbb{T}_i))$.

Mestre considered all prime levels up to 1429 and some higher ones. The dimension we find for the space $(\mathbb{T}(1) \otimes_\mathbb{Z} \mathbb{F}_2)/\tilde{\mathcal{R}}$ (see Prop. B.2.3) equals the dimension announced by Mestre. Moreover, he finds case $B(m)$ if and only if we find $m = \omega(\mathbb{T})$ (from the definition of the two numbers, the equality does not follow in general). We also calculated the image of the Galois representations associated to the eigenforms we found. These images agree with Mestre's claims. More precisely, we compute that for prime level N less than 2100 there exists an eigenform with image equal to $A_5 = \mathrm{SL}_2(\mathbb{F}_4)$ in the cases

$$N = 653, 1061, 1381, 1553, 1733, 2029$$

and equal to $\mathrm{SL}_2(\mathbb{F}_8)$ in the cases

$$N = 1429, 1567, 1613, 1693, 1997, 2017, 2089.$$

In all other prime cases, we find only dihedral images. However, we always find all the dihedral eigenforms predicted by the modified version of Serre's conjecture.

One of the main points of Mestre's letter to Serre was to conclude from the existence of an $\mathrm{SL}_2(\mathbb{F}_8)$-form that not all weight 1 forms arise as reductions of weight 1 forms from characteristic 0, even for an increased level because $\mathrm{SL}_2(\mathbb{F}_8)$ is not a quotient of a finite subgroup of $\mathrm{PGL}_2(\mathbb{C})$. We can reformulate that by saying that whenever there is an $\mathrm{SL}_2(\mathbb{F}_8)$-form, the space of Katz modular forms of weight 1 is strictly bigger than the space of classical forms.

To finish with, we wish to point out that in prime levels the representations associated to eigenforms of weight 1 in characteristic 2 were always found to be irreducible and the Hecke algebra to be of type Gorenstein. For non-prime square-free levels both properties can fail.

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