PLAN FOR THE THEORY OF REDUCTIVE GROUPS

Let F be a field. Let $\mathscr{C}_1$ be the category so that each object is a pair (G, T), where G is a connected reductive over $\bar{F}$ and T is a maximal torus of G and morphisms between (G, T) and (G', T') are *central isogenies* mapping T onto T'. Let $\mathscr{C}_2$ be the category whose objects are *root data* $(X, \phi, X^\vee, \phi^\vee)$ with ϕ a reduced root system and whose morphisms are *isogenies of root data* (see [Sp79, §1] for precise definition). Then there is a functor $f : \mathscr{C}_1 \to \mathscr{C}_2$ which associates to (G, T) a root datum $\Psi(G, T) = (X, \phi, X^\vee, \phi^\vee)$, where X and $X^\vee$ are respectively the character group and cocharacter group of T and ϕ and $\phi^\vee$ are respectively the roots and coroots of (G, T). Functor f has many nice properties [Sp79, Theorem 2.9]: dense, full, and not far from faithful. Therefore, category $\mathscr{C}_1$ is pretty much characterized by category $\mathscr{C}_2$.

Since Theorem 2.9 above has its origin from *Chevalley groups* (which are generated by root subgroups with explicit relations) and a proof of Theorem 2.9 in [Sp08, Ch. 9,10] does not look very attractive to me, it would be appropriate to study functor f in case G is a Chevalley group over $\bar{F}$. Therefore, the first two talks review complex semisimple Lie algebras and their representation theory which are essential to the constructions of Chevalley groups in the next two talks, the fifth talk relates Chevalley groups to semisimple algebraic groups and studies the properties of functor f for Chevalley groups. Then we review fundamentals of linear algebraic groups in the sixth talk and present root datum of reductive group, Theorem 2.9 and generalizations (p-morphisms) in the seventh talk. If time permits, we present F-forms of reductive groups in terms of Galois cohomology in the eighth talk and [Ti71] in the ninth talk.

1. $\mathfrak{sl}_{2,\mathbb{C}}$ and its representation theory. Sketch the general approach to complex semisimple Lie algebras: Cartan subalgebra $\to$ Cartan decomposition and roots $\to$ Weyl group and Killing form (present for example [FH91, Ch. 11,14]).
2. Weights and geometry of weights space, root system, classifications and Dynkin diagram, semisimplicity, irreducible representations and highest weights (present for example [FH91, Ch. 14,21]).
3. Chevalley basis (present [St68, §1, 2]).
4. Chevalley groups, relations among subgroups, Bruhat decomposition (present [St68, §3]).
5. Simplicity of Chevalley groups, relation with algebraic groups, study f for Chevalley groups (present [St68, §4, 5]).
6. Review fundamentals of linear algebraic groups prior to reductive groups (freestyle: present notation and theorems that you think are important).
7. Reductive group over $\bar{F}$: root datum, isomorphism theorem (present [Sp79, §1, 2]).
8. Review Galois cohomology and forms of reductive groups over F (present [Sp79, §3.1, 3.2, 3.3] and [Gi08, Prop. 29.4]).
9. Present the paper of Tits on irreducible F-representations of reductive group G over F [Ti71].

References

[FH91] W. Fulton and J. Harris: Representation Theory, Graduate Texts in Mathematics 129 (1st ed.), *Springer-Verlag* 1991.

[Gi08] P. Gille: Questions de rationalité sur les groupes algébriques linéaires, notes 2008.

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[Sp79] T.A. Springer: Reductive groups. Automorphic forms, representations and L-functions (Proc. Sympos. Pure Math., Oregon State Univ., Corvallis, Ore., 1977), Part 1, pp. 3–27, Proc. Sympos. Pure Math., XXXIII, Amer. Math. Soc., Providence, R.I., 1979.

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