

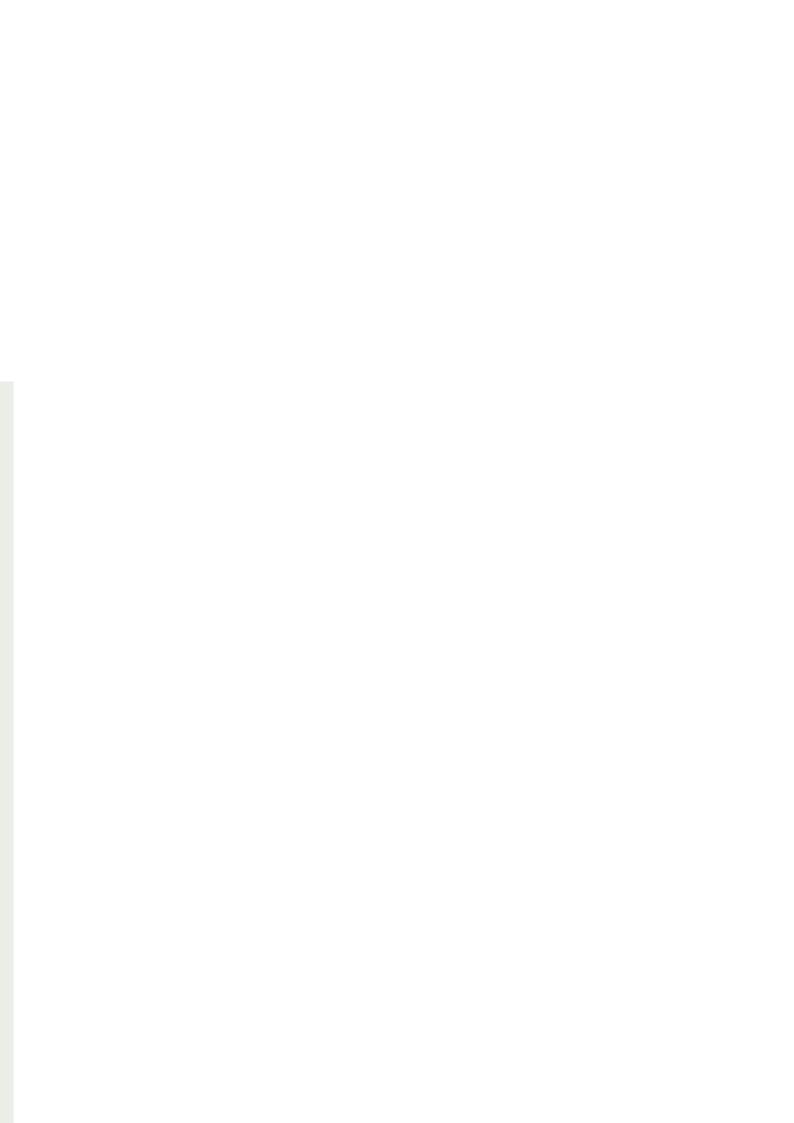
# 20 YEARS OF THE FOURTH MOMENT THEOREM

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# **About**

**Context.** The *Fourth Moment Theorem* (Nualart-Peccati, 2005) offers a remarkably simple criterion for Gaussian convergence on Wiener chaoses: for normalized multiple Wiener-Itô integrals, convergence in distribution to the standard normal is equivalent to the fourth moment tending to 3. This result has become a cornerstone of modern stochastic analysis and limit theorems.

Scope of the colloquium. Beyond celebrating this theorem, the colloquium offers an occasion to revisit twenty years of progress in stochastic analysis and asymptotic theory: universality phenomena on Wiener and Poisson chaoses; limit theorems for correlated structures and disordered systems; advances in nonlinear and singular stochastic PDEs; high-frequency asymptotics for random fields, random waves and spherical harmonics; and geometric aspects of probability, including functionals of Gaussian fields, stabilisation phenomena, and recent developments in stochastic geometry.

# **Programme**

## Thursday, December 11, 2025

13:00	Welcome and coffee
13:30	Martin Hairer (EPFL & Imperial College London): Title TBA
14:15	Maurizia Rossi (University of Milano-Bicocca): Some
	results for integral functionals of stationary Gaussian fields
15:00	Coffee break
15:30	Ciprian Tudor (University of Lille): Fourth Moment
	Theorem and Asymptotic Independence
16:15	Francesco Caravenna (University of Milano-Bicocca): On
	(ir)reducible Central Limit Theorems
17:00	Raphaël Lachièze-Rey (INRIA Paris): The Stein-Malliavin
	method for Poisson geometric functionals
20:30	Conference dinner

# Friday, December 12, 2025

09:00	Domenico Marinucci (University of Rome Tor Vergata): The
	Geometry of Random Neural Networks
09:45	Ronan Herry (University of Rennes): Stable Limits on
	Chaoses: Rates and Structure
10:30	Guillaume Poly (Nantes Université): Closability of
	Polynomial Chaos for Convergence in Distribution
11:15	Coffee break
11:45	Matthias Schulte (Hamburg University of Technology):
	Moderate deviations on Wiener and Poisson chaos
12:30	Giovanni Peccati (University of Luxembourg): The Fourth
	Moment Theorem: a Barcelona Adventure
12:55	David Nualart (Kansas University): Contributions to the
	Fourth Moment Theorem from Kansas
13:20	Lunch boxes
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# **Invited speakers**

Francesco Caravenna — University of Milano-Bicocca

Martin Hairer — EPFL and Imperial College London

Ronan Herry — University of Rennes

Raphaël Lachièze-Rey — INRIA Paris

Domenico Marinucci — University of Rome Tor Vergata

David Nualart — Kansas University

Giovanni Peccati — University of Luxembourg

Guillaume Poly — Nantes Université

Maurizia Rossi — University of Milano-Bicocca

Matthias Schulte — Hamburg University of Technology

Ciprian Tudor — University of Lille

# **List of Abstracts**

### **TBA**

### **Martin Hairer**

EPFL and Imperial College London

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### Some results for integral functionals of stationary Gaussian fields

### Maurizia Rossi

University of Milano-Bicocca

We investigate the asymptotic behavior of integral functionals of stationary Gaussian fields as the integration domain tends to be the whole space. More precisely, we establish a.s. and quantitative CLTs along with Malliavin regularity only under mild conditions on the covariance function of the underlying random field.

This talk is based on joint work with L. Maini and G. Zheng.

### **Fourth Moment Theorem and Asymptotic Independence**

### Ciprian Tudor

University of Lille

We will present recent results on the link between the Stein-Malliavin calculus, the Fourth Moment Theorem and the concept of asymptotic independence on Wiener chaos.

### On (ir)reducible Central Limit Theorems

### Francesco Caravenna

University of Milano-Bicocca

We present some Central Limit Theorems for homogeneous sums (or Wiener chaos) which have played an important role in the study of the 2D Stochastic Heat Equation and of 2D directed polymers in random environment. Motivated by these examples, we investigate whether Central Limit Theorems can be reduced to the classical Feller-Lindeberg setting of sums of independent random variables. This leads us to formulate conditions which ensure that such a reduction is not possible, expressed in terms of spectral properties of (hyper)graphs and of a certain notion of combinatorial dimension.

Based on joint works with F. Cottini, G. Peccati, R. Sun and N. Zygouras

### The Stein-Malliavin method for Poisson geometric functionals

### Raphaël Lachièze-Rey

**INRIA Paris** 

The study of asymptotic normality for geometric and topological statistics has advanced significantly since the foundational work of Bickel, Breiman, Kesten, Lee, and others in the late twentieth century. Landmark examples include nearest-neighbor statistics, minimal spanning tree functionals, Betti numbers, and related topological invariants. The field saw a major leap in the 2000s with Penrose and Yukich's generalization and strengthening of stabilization methods, particularly through martingale techniques. More recently, the Stein-Malliavin method on Poisson input has revolutionized the area, especially to obtain optimal speed of convergence, enabling new breakthroughs and ongoing progress.

This talk will survey the essential ideas behind these techniques, demonstrate their reach through representative examples, and discuss the latest developments.

### The Geometry of Random Neural Networks

### Domenico Marinucci

University of Rome Tor Vergata

We study the geometric properties of random neural networks by investigating their excursion sets for different activation functions, as the depth increases. We show that the boundary volumes exhibit fractal behavior for activations which are not very regular (e.g., the Heaviside step function), with their Hausdorff dimension monotonically increasing with the depth. On the other hand, for activations which are more regular (e.g., ReLU, logistic and tanh), as the depth increases the expected boundary volumes can either converge to zero, remain constant or diverge exponentially, depending on a single spectral parameter which can be easily computed. We then discuss central and non-central limit theorems for nonlinear functionals defined on the excursion sets; in particular, we show that the limiting distribution depends on the same spectral parameter. Our theoretical results are confirmed in some numerical experiments based on Monte Carlo simulations.

Based on joint works with Simmaco Di Lillo, Leonardo Maini, Michele Salvi and Stefano Vigogna.

**Stable Limits on Chaoses: Rates and Structure** 

**Ronan Herry** 

University of Rennes

In this talk, I will present a panoramic overview of convergence in law on the Wiener space. Starting from the celebrated  $\Gamma$ -Stein bound: a sequence  $(F_n)$  of random variables converges in law to a normal distribution if (and only if)  $\Gamma(F_n, L^{-1}F_n)$  converges to a constant, I will present variations around this approach.

1. Super-convergence results, showing that convergence in law automatically entails uniform convergence of the densities together with that of all their derivatives.

2. New characterisation of CLTs on Wiener chaoses.

3.  $\Gamma$ -Stein results beyond the Gaussian setting where the Markov generator is non-invertible, yielding sharp rates of convergence for certain interacting particles models.

Along the way, I will highlight two key ingredients: the Bouleau derivative, also known as the sharp operator; and the directional influence.

This talk constitutes the first part of a two-part presentation with Guillaume Poly, who will build upon these ideas to show that Wiener chaoses are closed for the convergence in law.

The results presented are based on a series of joint works with Jürgen Angst, Dominique Malicet and Guillaume Poly.

### **Closability of Polynomial Chaos for Convergence in Distribution**

### **Guillaume Poly**

Nantes Université

In this talk, we study general polynomial chaoses—multivariate polynomials evaluated at random variables—and characterize their closure in distribution. In the special case of Gaussian polynomial chaoses, this characterization settles a longstanding open problem dating back to the seminal works of Kolmogorov and Sevastyanov in the 1960s. We will examine several applications and related conjectures in depth, and we will highlight the role of various criteria for central convergence in Wiener chaoses used in our arguments.

The content of the talk is based on a joint work with R.Herry and D.Malicet.

### **Moderate deviations on Wiener and Poisson chaos**

### **Matthias Schulte**

Hamburg University of Technology

For multiple Wiener-Itô integrals with respect to isonormal Gaussian processes or Poisson processes we consider moderate deviation principles as well as concentration inequalities and normal approximation bounds with Cramér correction. In the Gaussian case, we obtain the same conditions as in the fourth moment theorem. The results rely on the method of cumulants, which requires sharp estimates on cumulants. Applications of the general results will be discussed.

This talk is based on joint work with Christoph Thäle.

### The Fourth Moment Theorem: a Barcelona Adventure

### Giovanni Peccati

University of Luxembourg

In this talk, I will briefly recall the main motivations behind the first formulation and proof of the Fourth Moment Theorem, and describe how the correct strategy gradually emerged from an intense month of discussions that David and I shared while I was visiting the University of Barcelona in February 2003.

### **Contributions to the Fourth Moment Theorem from Kansas**

### **David Nualart**

University of Kansas

In this talk, I will describe a few applications and extensions of the Fourth Moment Theorem, that were done while I was visiting the University of Kansas, thanks to an invitation by Yaozhong Hu.

# **Participants**

Michael Althof LEIW Management

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