20 YEARS OF THE FOURTH MOMENT THEOREM

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Martin Hairer
Ronan Herry
Raphaël Lachièze-Rey
Domenico Marinucci
David Nualart
Giovanni Peccati
Guillaume Poly
Maurizia Rossi
Matthias Schulte
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UNIVERSITY OF LUXEMBOURG

Contents

About

Context. The *Fourth Moment Theorem* (Nualart–Peccati, 2005) offers a remarkably simple criterion for Gaussian convergence on Wiener chaoses: for normalized multiple Wiener–Itô integrals, convergence in distribution to the standard normal is equivalent to the fourth moment tending to 3. This result has become a cornerstone of modern stochastic analysis and limit theorems.

Scope of the colloquium. Beyond celebrating this theorem, the colloquium offers an occasion to revisit twenty years of progress in stochastic analysis and asymptotic theory: universality phenomena on Wiener and Poisson chaoses; limit theorems for correlated structures and disordered systems; advances in nonlinear and singular stochastic PDEs; high-frequency asymptotics for random fields, random waves and spherical harmonics; and geometric aspects of probability, including functionals of Gaussian fields, stabilisation phenomena, and recent developments in stochastic geometry.

Programme

Thursday, December 11, 2025

13:00	Welcome and coffee
13:30	**Martin Hairer** (EPFL & Imperial College London): *The variance blow-up phenomenon*
14:15	**Maurizia Rossi** (University of Milano–Bicocca): *Some results for integral functionals of stationary Gaussian fields*
15:00	Coffee break
15:30	**Ciprian Tudor** (University of Lille): *Fourth Moment Theorem and Asymptotic Independence*
16:15	**Francesco Caravenna** (University of Milano–Bicocca): *On (ir)reducible Central Limit Theorems*
17:00	**Raphaël Lachièze-Rey** (INRIA Paris): *The Stein-Malliavin method for Poisson geometric functionals*
20:30	Conference dinner

Friday, December 12, 2025

09:00	**Domenico Marinucci** (University of Rome Tor Vergata): *The Geometry of Random Neural Networks*
09:45	**Ronan Herry** (University of Rennes): *Stable Limits on Chaoses: Rates and Structure*
10:30	**Guillaume Poly** (Nantes Université): *Closability of Polynomial Chaos for Convergence in Distribution*
11:15	Coffee break
11:45	**Matthias Schulte** (Hamburg University of Technology): *Moderate deviations on Wiener and Poisson chaos*
12:30	**Giovanni Peccati** (University of Luxembourg): *The Fourth Moment Theorem: a Barcelona Adventure*
12:55	**David Nualart** (Kansas University): *Contributions to the Fourth Moment Theorem from Kansas*
13:20	Lunch boxes

Invited speakers

Francesco Caravenna — University of Milano–Bicocca
Martin Hairer — EPFL and Imperial College London
Ronan Herry — University of Rennes
Raphaël Lachièze-Rey — INRIA Paris
Domenico Marinucci — University of Rome Tor Vergata
David Nualart — Kansas University
Giovanni Peccati — University of Luxembourg
Guillaume Poly — Nantes Université
Maurizia Rossi — University of Milano–Bicocca
Matthias Schulte — Hamburg University of Technology
Ciprian Tudor — University of Lille

List of Abstracts

The variance blow-up phenomenon

Martin Hairer

EPFL and Imperial College London

Some results for integral functionals of stationary Gaussian fields

Maurizia Rossi

University of Milano–Bicocca

We investigate the asymptotic behavior of integral functionals of stationary Gaussian fields as the integration domain tends to be the whole space. More precisely, we establish a.s. and quantitative CLTs along with Malliavin regularity only under mild conditions on the covariance function of the underlying random field.

This talk is based on joint work with L. Maini and G. Zheng.

Fourth Moment Theorem and Asymptotic Independence

Ciprian Tudor

University of Lille

We will present recent results on the link between the Stein-Malliavin calculus, the Fourth Moment Theorem and the concept of asymptotic independence on Wiener chaos.

On (ir)reducible Central Limit Theorems

Francesco Caravenna

University of Milano–Bicocca

We present some Central Limit Theorems for homogeneous sums (or Wiener chaos) which have played an important role in the study of the 2D Stochastic Heat Equation and of 2D directed polymers in random environment. Motivated by these examples, we investigate whether Central Limit Theorems can be reduced to the classical Feller-Lindeberg setting of sums of independent random variables. This leads us to formulate conditions which ensure that such a reduction is not possible, expressed in terms of spectral properties of (hyper)graphs and of a certain notion of combinatorial dimension.

Based on joint works with F. Cottini, G. Peccati, R. Sun and N. Zygouras

The Stein-Malliavin method for Poisson geometric functionals

Raphaël Lachièze-Rey

INRIA Paris

The study of asymptotic normality for geometric and topological statistics has advanced significantly since the foundational work of Bickel, Breiman, Kesten, Lee, and others in the late twentieth century. Landmark examples include nearest-neighbor statistics, minimal spanning tree functionals, Betti numbers, and related topological invariants. The field saw a major leap in the 2000s with Penrose and Yukich's generalization and strengthening of stabilization methods, particularly through martingale techniques. More recently, the Stein-Malliavin method on Poisson input has revolutionized the area, especially to obtain optimal speed of convergence, enabling new breakthroughs and ongoing progress.

This talk will survey the essential ideas behind these techniques, demonstrate their reach through representative examples, and discuss the latest developments.

The Geometry of Random Neural Networks

Domenico Marinucci

University of Rome Tor Vergata

We study the geometric properties of random neural networks by investigating their excursion sets for different activation functions, as the depth increases. We show that the boundary volumes exhibit fractal behavior for activations which are not very regular (e.g., the Heaviside step function), with their Hausdorff dimension monotonically increasing with the depth. On the other hand, for activations which are more regular (e.g., ReLU, logistic and $\tanh$), as the depth increases the expected boundary volumes can either converge to zero, remain constant or diverge exponentially, depending on a single spectral parameter which can be easily computed. We then discuss central and non-central limit theorems for nonlinear functionals defined on the excursion sets; in particular, we show that the limiting distribution depends on the same spectral parameter. Our theoretical results are confirmed in some numerical experiments based on Monte Carlo simulations.

Based on joint works with Simmaco Di Lillo, Leonardo Maini, Michele Salvi and Stefano Vigogna.

Stable Limits on Chaoses: Rates and Structure

Ronan Herry

University of Rennes

In this talk, I will present a panoramic overview of convergence in law on the Wiener space. Starting from the celebrated Γ-Stein bound: a sequence (F_n) of random variables converges in law to a normal distribution if (and only if) $\Gamma(F_n, L^{-1}F_n)$ converges to a constant, I will present variations around this approach.

1. Super-convergence results, showing that convergence in law automatically entails uniform convergence of the densities together with that of all their derivatives.

2. New characterisation of CLTs on Wiener chaoses.

3. Γ-Stein results beyond the Gaussian setting where the Markov generator is non-invertible, yielding sharp rates of convergence for certain interacting particles models.

Along the way, I will highlight two key ingredients: the Bouleau derivative, also known as the sharp operator; and the directional influence.

This talk constitutes the first part of a two-part presentation with Guillaume Poly, who will build upon these ideas to show that Wiener chaoses are closed for the convergence in law.

The results presented are based on a series of joint works with Jürgen Angst, Dominique Malicet and Guillaume Poly.

Closability of Polynomial Chaos for Convergence in Distribution

Guillaume Poly

Nantes Université

In this talk, we study general polynomial chaoses—multivariate polynomials evaluated at random variables—and characterize their closure in distribution. In the special case of Gaussian polynomial chaoses, this characterization settles a longstanding open problem dating back to the seminal works of Kolmogorov and Sevastyanov in the 1960s. We will examine several applications and related conjectures in depth, and we will highlight the role of various criteria for central convergence in Wiener chaoses used in our arguments.

The content of the talk is based on a joint work with R.Herry and D.Malicet.

Moderate deviations on Wiener and Poisson chaos

Matthias Schulte

Hamburg University of Technology

For multiple Wiener-Itô integrals with respect to isonormal Gaussian processes or Poisson processes we consider moderate deviation principles as well as concentration inequalities and normal approximation bounds with Cramér correction. In the Gaussian case, we obtain the same conditions as in the fourth moment theorem. The results rely on the method of cumulants, which requires sharp estimates on cumulants. Applications of the general results will be discussed.

This talk is based on joint work with Christoph Thäle.

The Fourth Moment Theorem: a Barcelona Adventure

Giovanni Peccati

University of Luxembourg

In this talk, I will briefly recall the main motivations behind the first formulation and proof of the Fourth Moment Theorem, and describe how the correct strategy gradually emerged from an intense month of discussions that David and I shared while I was visiting the University of Barcelona in February 2003.

Contributions to the Fourth Moment Theorem from Kansas

David Nualart

University of Kansas

In this talk, I will describe a few applications and extensions of the Fourth Moment Theorem, that were done while I was visiting the University of Kansas, thanks to an invitation by Yaozhong Hu.

Participants

Hamza Aaboud	University of Luxembourg
Michael Althof	LEIW Management
Marwa Banna	NYU Abu Dhabi
Yannick Baraud	University of Luxembourg
Jean-Marc Bardet	Paris 1 Panthéon–Sorbonne University
Samir Ben Hariz	Le Mans University
Felix Benning	University of Luxembourg
Gaspard Bernard	University of Luxembourg
Yanis Bosch	University of Twente
Solesne Bourguin	Boston University
Francesco Caravenna	University of Milano–Bicocca
Lucia Celli	University of Luxembourg
Lorenzo Cristofaro	University of Luxembourg
Mauricio Daros Andrade	The School of Applied Mathematics at FGV
Thomas Devos	University of Liège
Julie Gamain	ESI Léonard de Vinci
Ujan Gangopadhyay	University of Luxembourg
Martin Hairer	EPFL and Imperial College London
Akram Heidari	University of Luxembourg
Ronan Herry	University of Rennes
Christian Houdré	Georgia Institute of Technology
Bingyu Jiang	University of Luxembourg
Tom Klose	University of Oxford
Sefika Kuzgun	MPI for Mathematics in the Sciences
Raphaël Lachièze-Rey	INRIA Paris
Christophe Ley	University of Luxembourg
Sophia Loizidou	University of Luxembourg
Laurent Loosveldt	University of Liège

Luis da Maia	University of Luxembourg
Paul Mansanarez	Université Libre de Bruxelles
Domenico Marinucci	University of Rome Tor Vergata
Yassine Nachit	University of Lille
Ivan Nourdin	University of Luxembourg
David Nualart	Kansas University
Chiara Passamonti	University of Luxembourg
Giovanni Peccati	University of Luxembourg
Pierre Perruchaud	University of Luxembourg
Francisco Pina	University of Luxembourg
Francesca Pistolato	University of Luxembourg
Mark Podolskij	University of Luxembourg
Gregorio Poggi	University of Padova
Guillaume Poly	Nantes Université
Gabriel Romon	University of Luxembourg
Maurizia Rossi	University of Milano–Bicocca
Giacomo Salvati	University of Luxembourg
Matthias Schulte	Hamburg University of Technology
Michele Stecconi	University of Luxembourg
Grégoire Szymanski	University of Luxembourg
Tara Trauthwein	University of Münster
Ciprian Tudor	University of Lille
Xiaochuan Yang	Banque Internationale à Luxembourg
Liqian Zhang	University of Manchester
Pavlos Zoubouloglou	University of Münster
Jérémy Zurcher	Georgia Institute of Technology

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