PhD Days 2017: Abstracts

Friday 29th

Gilles Becker: ***Schwartz spaces on quotients*** $\Gamma\backslash G$ ***of Lie groups by discrete subgroups***

In the first part of the talk, we provide several examples of quotients $\Gamma\backslash G$ of Lie groups by discrete subgroups. In the second part, we introduce convex-cocompact, noncocompact, torsion-free subgroups of G (here G is a real simple connected Lie groups of real rank one) and we define the Schwartz space $\mathscr{C}(\Gamma\backslash G)$ for these discrete subgroups.

Jimmy Devillet: ***On quasitrivial symmetric nondecreasing associative operations***

Let X be a nonempty set and let $n \geq 2$ be an integer. The n-ary operations $F : X^n \to X$ satisfying the associativity property have been extensively investigated since the pioneering work by Dörnte and Post. In the algebraic language, when F is such an operation, the pair (X, F) is called an n-ary semigroup.

In this talk we investigate the class of n-ary operations $F : X^n \to X$ on a linearly ordered set that are quasitrivial, symmetric, nondecreasing and associative (quasitriviality means that F always outputs one of its input values). After presenting some definitions we provide a characterization of these operations and show that they are derived from associative binary operations. Finally, we provide an alternative characterization of these operations in terms of single-peaked linear orderings.

Massimo Notarnicola: ***Introduction to random variables and an example of random walk***

In this talk we define the basic notions of random variable and probability measure. We then provide a concrete example of a random walk: the gamblers ruin. At each game, a gambler starting with initial fortune k (euros), either wins 1 euro or loses 1 euro with prescribed probabilities. His aim is to attain a total fortune of$K > k$ euros before running out of money. The game stops in both situations. We are interested in the computation of the probability that the gambler is ruined before winning.

Anna Vidotto: ***An introduction to Gaussian approximation via simple examples***

Starting with a very simple example, we will see the extent and importance of Gaussian approximation in applications. In particular, beginning with the most ordinary formulation of Central Limit Theorem (CLT), we will extend it to more complicated situations. Indeed, always referring to some easy and classical examples, we will try to get closer to some of the latest research streams in Gaussian approximation.

Ronan Herry: ***An introduction to Gaussian measures***

I will introduce the Gaussian distributions on the line and explain some of the main analytic-probabilistic properties. I will then discuss the multivariate generalisation and the infinite-dimensional generalisation through the formalism of the Gaussian Hilbert space. I will give some applications to illustrate the power of this formalism. All measure theoretic and probabilistic notions shall be recalled.

Robert Baumgarth: ***Stochastic Differential Geometry***

We introduce diffusions on smooth Riemannian manifolds. The most well-studied example is Brownian motion on (smooth Riemannian) manifolds. The deep connection between the Laplacian as being the generator of Brownian motion (up to a constant) has led to a completely new research area called *Stochastic Differential Geometry*, i.e. the stochastic analysis on manifolds. Stochastic methods open ways to solve analytic and geometric problems in much more elegant fashions. Hence, we discuss very briefly modern notions and concepts of this relatively young and interdisciplinary subject. All notions will be briefly introduced during the talk as needed concerning the broad audience.

Diu Tran: ***Statistical inference for Hermite Ornstein-Uhlenbeck processes***

Let $Z^{q,H}$ be a Hermite process of order $q \geq 1$ and Hurst parameter $H \in (1/2, 1)$. It is a self-similar stochastic process with long-range dependence. It includes the celebrated fractional Brownian motion. We consider the following SDE driven by Hermite process as a noise:

$$dX_t = -\alpha X_t dt + dZ_t^{q,H}$$

The strong unique solution of this SDE is called Hermite Ornstein-Uhlenbeck process. In this talk, we will deal with the problem about construction estimator for drift parameter α. A strong consistent estimator in the sense of almost sure convergence is found. The rate of convergence of the estimator is established.

Saturday 30th

Assar Anderson: *A non-technical introduction to operads*

Operads are objects that encodes different types of algebraic structures. They first emerged in the 1960's, as a tool in algebraic topology, with the final definition and the word "operad" due to Jon Peter May. Since then they have appeared in various fields of mathematics.

The aim of this talk is to explain the ingredients of an operad, what a structures they can encode, and how we can think of operads in an intuitive way.

Jill Ecker: *Low-Dimensional Cohomology of the Witt and the Virasoro Algebra*

The aim of our work is to study the low-dimensional cohomology groups of the Witt and the Virasoro algebra from a purely algebraic viewpoint. This presentation is based on 1707.06106 [math.RA], which is joint work with Martin Schlichenmaier. The talk starts with a brief introduction of the Witt algebra and its central extension, the Virasoro algebra. In a second step, the Chevalley-Eilenberg cohomology of Lie algebras is described, with a particular focus on low-dimensional cohomology groups with values in the adjoint module. We will illustrate our algebraic techniques on the proof of the vanishing of the first cohomology group with values in the adjoint module of the Witt algebra. We will finish by commenting on the proof of the same result for the third cohomology group. The proof is a generalization of the one developed by Schlichenmaier to prove the vanishing of the second cohomology group.

Andrea Tamburelli: *Hyperbolic affine spheres and convex real projective structures*

Affine geometry studies curves and surfaces in the Euclidean space, up to affine transformations. The usual quantities we are familiar with, like curvature and metric, are no longer well defined. In this talk, I will introduce the notion of hyperbolic affine spheres, which form a particular class of surfaces, whose immersions are completely classified. We then see how these objects are related to representations in $\mathrm{SL}(3, R)$ and convex sets in the projective plane.

Filippo Mazzoli: *Classical applications of the H-cobordism theorem*

n this talk I will introduce the H-cobordism theorem, following the paper *Lectures on the H-Cobordism Theorem* by Milnor, John, et al. (Princeton University Press, 1965. JSTOR, `www.jstor.org/stable/j.ctt183psc9`). My purpose will be to describe the statement of the theorem and of some very classical tools of algebraic topology which will allow us to show some simple but very powerful applications, as the characterisation of the discs and the generalised Poincars Conjecture in dim > 4.

Luca Notarnicola: *The Arithmetic of Ellipitic Curves*

The study of elliptic curves has always been an important task in Number Theory and many topics widely rely on it. For instance, Gerhard Freys approach to prove Fermats Last Theorem makes use of a special elliptic curve. The goal of my talk is to introduce some basics of elliptic curves. Roughly spoken, elliptic curves over a given field are smooth algebraic projective plane curves of genus one and together with a special point $\mathcal{O}$, called the point at infinity. What makes them special is that they define group varieties with neutral element $\mathcal{O}$. Further, it is worth studying the natural Galois representations that come with elliptic curves. I will conclude my talk by introducing the L-series associated to an elliptic curve.

Emiliano Torti: ***Modular Forms***

This talk is a part of a series aimed to present the key steps in the proof of Fermat's Last theorem. In this short talk, we will introduce the notion of modular form through different equivalent definitions. We will focus on their role of interplay between several areas of mathematic such as Algebraic and Differential geometry, Complex analysis and Algebra. Finally, we will associate to each modular form a Galois representation. These modular Galois representation together with the ones associated to an Elliptic curve will play a central role in the proof of Fermat's Last theorem.

Mariagiulia De Maria: ***Fermat's Last Theorem and the Universe of Number Theory***

What is today know as Fermat's Last Theorem was just a little note the Fermat scrabbled on the margin of one of his books in the 1630s. While almost all his other annotations were proven during the 18th century, his assertion "*On the other hand, it is impossible to separate [...] any power except a square into two powers with the same exponent.*"was not as easily proven. Finally, a complete proof of this theorem was presented by Andrew Wiles in 1994, centuries after the formulation.
In this talk, we will present the history of this theorem together with the main ingredients to his demonstration. We will show how elliptic curves and modular forms play an important role in the proof and in particular how they are related to each other. This will give us the possibility to take a look at all the objects that are studied in Number Theory and their correlation to each other.