Gaps between Pythagorean triples

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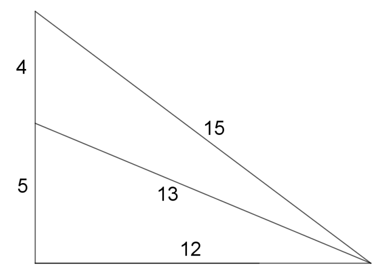
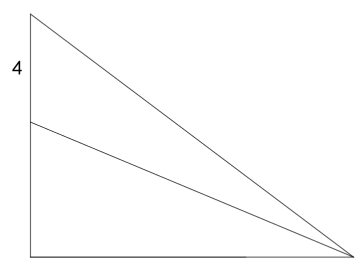
Pythagorean triples are triples of integers (a, b, c) that satisfy the equation $a^2 + b^2 = c^2$. They form the side lengths of a right angled triangle and have both applications to geometry and number theory. Examples are

$$(3,4,5), \quad (6,8,10), \quad (5,12,13), \quad (9,12,15), \quad (8,15,17), \quad \ldots$$

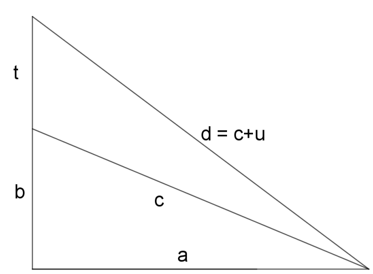
It is possible to parametrise the set of all Pythagorean triples, but there are still many questions about them that this parametrisation does not answer. For example, from the above list, you may notice that one can draw the following picture: One may ask the question of whether this situation is unique, i.e. if there are



more pictures you can draw in which one side an integer sided right angled triangle is extended by 4 to yield another integer sided right angled triangle:



Or more generally, can we classify all the integer sided right angled triangles which fit into a picture of the following shape: This project aims to explore these and related questions to create pictures, prove



algebraic results about when this may or may not be possible, and possibly even analytic results about how often these types of pictures can be drawn. A partial resolution of this question can be found here:

`https://math.stackexchange.com/questions/4645685/gap-between-two-pythagorean-triples/4647611#4647611`

Prerequisites are basic algebra (groups and rings) and a motivation to learn about number theory. Basic knowledge of visualisation software like Geogebra or coding languages like Python is encouraged but not required. The participating students will learn about Pell's equation and continued fractions and how to implement these classical notions from number theory effectively into a computer.

For more reading about ways to visualise Pythagorean triples, please see here:

`https://www.researchgate.net/publication/2440825_The_Modular_Tree_of_Pythagoras`