# Kakeya Needle Problem: Numerical Constructions and Experimental Investigations

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The Kakeya needle problem asks whether it is possible to rotate a unit line segment fully within a planar set of arbitrarily small area. This classical question lies at the intersection of analysis, geometry, fractal theory, and number theory. While the full conjecture is deep and still open in many settings, approximate or discrete constructions allow for computational experiments and visualizations. In this project, students will implement algorithms to generate Kakeya-type sets, explore their geometric properties, and illustrate how a line segment can rotate inside them. They will also investigate finite approximations and fractal-like behavior through computational experiments.

## Goals

Possible directions and goals include:

- Implement simple discrete models of Kakeya sets on a pixel grid and produce visualizations/animations of a segment rotating within them.
- Reproduce classical constructions (e.g., Besicovitch sets) in a discretized setting.
- Measure and compare the area and scaling properties of different constructions.
- Estimate fractal dimensions (box-counting dimension) of discrete approximations.
- (Optional, advanced) Explore finite-field versions of the Kakeya problem and compare numerical results with known theoretical bounds.

## Tools / Prerequisites

- **Tools:** Python (NumPy, Matplotlib, possibly Plotly for interactive visualizations), SageMath (for computations over finite fields).
- **Prerequisites:** Basic linear algebra and analysis; programming experience in Python; some familiarity with fractals or measure theory is a plus but not required.

## Links

- Wikipedia on Kakeya sets.
- Needle shown rotating inside a deltoid.
- Kakeya Needle Problem Animation.

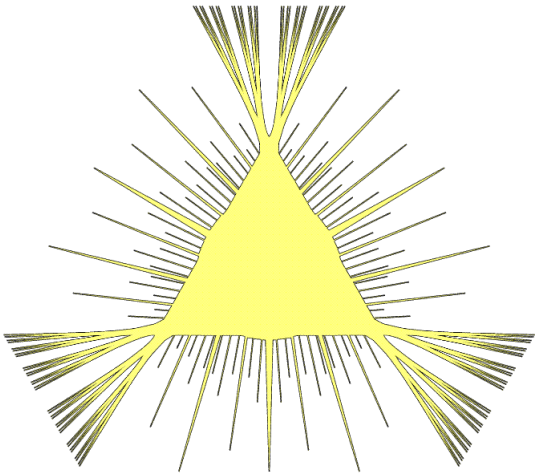

Figure 1: A Kakeya needle set constructed from Perron trees.