The Fibonacci Word Fractal: Construction and Geometric Properties

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The Fibonacci word fractal is a geometric object obtained by iteratively interpreting the Fibonacci word (a well-known sequence from combinatorics on words) as drawing instructions. The resulting curve exhibits self-similarity and complex fractal structure. The project consists of constructing this fractal computationally and exploring its mathematical properties, with a special focus on its fractal dimension and the geometry of its boundary. As an extension, students may also investigate *Fibonacci tilings*, which are closely related to the same combinatorial structures and lead to rich geometric and dynamical behavior.

Goals

Possible directions and goals include:

- Implement algorithms to generate the Fibonacci word fractal and produce high-quality visualizations.
- Estimate numerically the fractal (Hausdorff or box-counting) dimension of the curve and its boundary.
- Compare computational results with known theoretical bounds or conjectures.
- (Optional) Explore connections to Fibonacci tilings and investigate their geometric properties.
- Produce animations illustrating the iterative construction of the fractal curve.

Tools / Prerequisites

- **Tools:** Python (NumPy, Matplotlib, Plotly for interactive graphics), SageMath (for combinatorial generation of Fibonacci words), image-processing libraries for dimension estimation.

- **Prerequisites:** Programming experience in Python; some familiarity with box-counting dimension or fractal geometry is helpful but not required.

Links

- Wikipedia on Fibonacci word fractal.
- There is abundant literature on the subject.

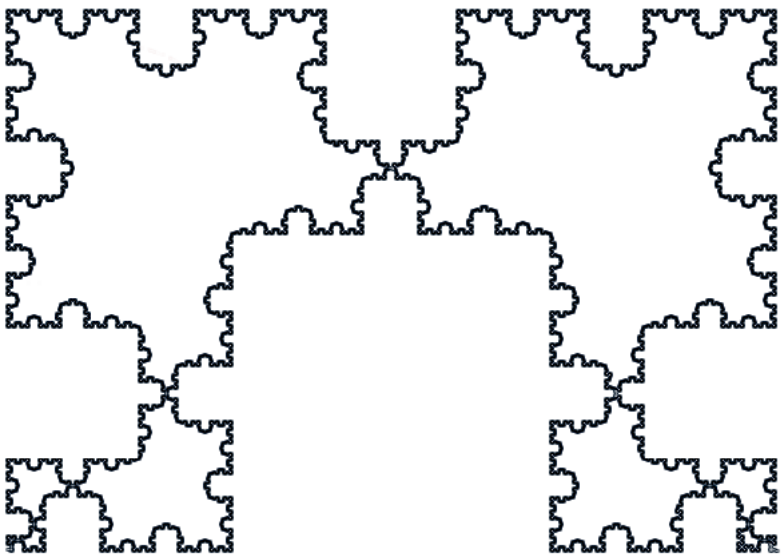

Figure 1: Fibonacci word fractal boundary.