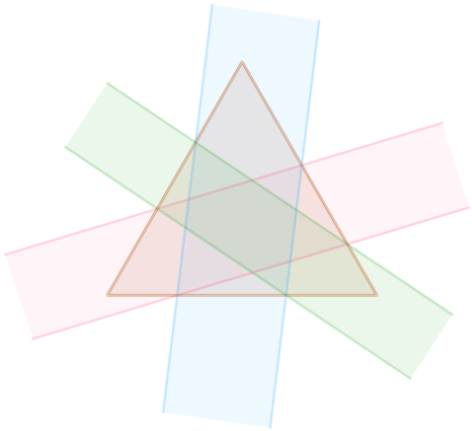
Plank problem

EML project, winter semester 2025

Field: Geometry	Means of experimentation: Python + LLMs
Language: English	Supervisor: Alexey Balitskiy
Difficulty: adjustable	Contact: alexey.balitskiy@uni.lu

About the problem



You are given a convex shape in the plane, and it is covered by a few planks (or strips), like in this picture ⟶

Question: what is the minimal total width (sum of the widths) of the planks needed for this?

The difficulty of this problem varies greatly depending on how we measure widths. If for each plank we measure its usual width, then Thøger Bang proved [1] that their total width cannot be less than if we cover our shape with just one plank as narrow as possible.

If instead we measure widths of each plank *relative to the width of the plank of the same direction covering our shape*, then Bang conjectured that their sum is at least 1, but we only know this for centrally symmetric shapes [2]. Can you compute the sum of three relative widths in the example in the picture?

Goals

The problem is open even for the case of three planks, so we will restrict to the following question: *characterize equality cases in Bang's conjecture for three-plank coverings.* To this end, we will

- write and debug code (perhaps in Python, perhaps with LLM assistants) generating covers,
- visualize them,
- interpret pictures and make conjectures based on what we see!

For the last part you'll need mathematical curiosity, which cannot be replaced by an LLM as of now.

References

[1] Th. Bang. "A solution of the Plank problem." Proceedings of the American Mathematical Society 2.6 (1951): 990-993.

[2] K. Ball. "The plank problem for symmetric bodies." Inventiones mathematicae 104.1 (1991): 535-543.