

Project Proposal — Experimental Mathematics Lab
Rivin’s Rationality Conjecture for Polyhedra

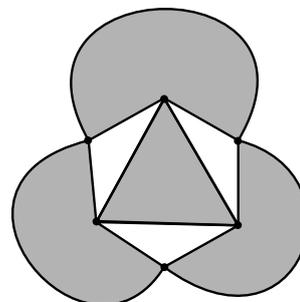
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Goals. The aim of this project is to provide new insights into a recent conjecture on the dihedral angles of ideal polyhedra of maximum volume [1]. Though the original question stems from hyperbolic geometry, this project will explore a completely new approach (see context below) only relying on Euclidean geometry. The main goals will be to

- (1) Create a numerical framework to find *maxvol-structures* for given combinatorics.
- (2) Building on this framework to explore the properties of *maxvol-structures*. In particular, we are interested in better understanding the question introduced below.

Context. We start with an abstract blueprint for the combinatorics of our *maxvol-structure*. Consider a simple planar graph $\mathcal{G} = (V, E)$, *i.e.*, a graph that can be drawn in the plane without self-intersections (see inset). It divides the plane into connected regions—one of which is unbounded. We call these regions *faces* and denote the set of faces by F . We impose further conditions on \mathcal{G} :

- (1) It is *face-bipartite*, *i.e.*, the set of faces F can be split into two disjoint subsets F_\bullet and F_\circ , such that each face of F_\bullet only shares edges with faces in F_\circ and vice versa;
- (2) All faces in F_\circ are *triangles*, *i.e.*, they are incident to exactly three edges;
- (3) All vertices in V have degree 4.



The set of *corners* $\mathcal{C}_{\mathcal{G}}$ of \mathcal{G} is the set of pairs (f, v) where f is a face adjacent to the vertex v . It follows from condition (3) that each corner (f, v) possesses an opposite corner (f', v) , *i.e.*, f and f' share a common vertex v but no common edge. Given this combinatorial setup, a *maxvol-structure* on \mathcal{G} is an assignment of angles to the corners of the faces, *i.e.*, a function $\theta: \mathcal{C}_{\mathcal{G}} \rightarrow (0, \pi)$, such that:

- (1) Each face can be realized as a Euclidean convex polygon with the prescribed angles and all edges of length 1;
- (2) Opposite corners are assigned the same angle.

Question. Let (\mathcal{G}, θ) be a *maxvol-structure*. Are all angles of the faces in F_\bullet integer multiples of $2\pi/n$, where n is the least common multiple (lcm) of the degrees of the faces in F_\bullet ?

Prerequisites.

- Basic familiarity with Euclidean geometry;
- Programming experience (e.g., in *python*).

Supervisors.

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References

- [1] Rivin, Igor. *Maximal Volume Ideal Polyhedra and the Arithmetic Angle Phenomenon*. 2025. arXiv: 2512.10087