# No three in a line: the geometry of SET

The game SET consists of 81 cards: each described by 4 attributes (number, color, shape, shading), each taking 3 possible values. A valid play consists of finding 3 cards such that for each attribute, the three values are either all equal or all different. A natural question arises: what is the minimum number of cards on the table to guarantee that a valid play exists among them?

This question has a clean mathematical formulation. Identifying each card with a vector in $(\mathbb{F}_3)^4$—where $\mathbb{F}_3 = \mathbb{Z}/3\mathbb{Z}$ is the field with 3 elements—a valid play is precisely a *line*. The question then becomes: what is the minimum number $C$ such that any subset $A \subseteq (\mathbb{F}_3)^4$ with $|A| > C$ is guaranteed to contain a line? Equivalently, what is the maximum size of a *cap set*, i.e., a subset containing no line?

The goals of this project are:

- To implement an algorithm detecting lines in subsets of $(\mathbb{F}_3)^4$ and to determine the maximum size of a cap set in $(\mathbb{F}_3)^4$ by computational methods.
- To explore the behavior of cap sets in $(\mathbb{F}_3)^n$ for $n > 4$: search for large cap sets computationally, look for patterns, and derive upper and lower bounds.
- To generalize the problem to $(\mathbb{F}_p)^n$ for primes $p > 3$ and search for similar invariants.

**Prerequisites:** Basic modular arithmetic and linear algebra. Familiarity with Python is helpful but not strictly required.

**Supervisor:** Javier Fernández Píriz, contact: javier.fernandezpiriz@uni.lu