Experimenting with AI to study open questions in geometry

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We list below a series of open questions in geometry that have elementary statements and can be tackled with the help of AI systems. AI can be a great help to search for counter-examples directly, or to write a program to search a wide space of possibilities for counter-examples. In some cases it can also help obtain proofs.

The problems listed below are provided *as examples only.* They are not a closed list, and they are not ranked. Other problems are perfectly acceptable — in discrete geometry, in convex geometry, in combinatorics, in number theory, or anywhere else — and a problem that you find yourself, or that comes out of your own reading, is usually a better choice than one taken off a list.

The point of the exercise is not necessarily to solve a famous open problem. That is unlikely, and it is not the criterion of success. The point is to **learn to use AI tools in a rigorous and creative way to study a mathematical problem**.

Rigorous means: never call a statement a theorem unless you have a complete proof of it. If a step is missing, say so explicitly, call the statement a conjecture, and state precisely what would have to be proved to turn it into a theorem. An AI system will produce fluent, confident and wrong arguments at any moment, and the failure is invisible unless you have an independent check. The deliverable of a project should therefore be a *certificate*: an UNSAT proof log validated by an independent checker, an interval-arithmetic computation with a certified enclosure, an explicit object verified by a short independent script, or a formal proof in Lean. "The model explained why it is true" is not a result.

Creative means: the interesting work is upstream of the computation. Reformulating the question so that it becomes finite; choosing an encoding; deciding which symmetries to break; deciding what to search for, and what a negative answer would tell you. A partial result — a better encoding, a ruled-out special case, a new record configuration — is a genuine contribution and a perfectly good outcome.

For most of the problems below, the realistic short-term target is a well-defined sub-problem, not the conjecture itself. Identifying that sub-problem is part of the work.

A list of possible problems

1. A 6-chromatic unit-distance graph (Hadwiger–Nelson). The chromatic number of the plane — the least number of colours needed so that points at distance 1 receive different colours — is known to be 5, 6 or 7. The lower bound 5 comes from de Grey's 2018 finite unit-distance graph, later reduced in size by several authors. A 6-chromatic finite unit-distance graph would be a finite object, verifiable by a SAT solver in seconds; the whole difficulty is the search, over configurations built from rotated copies of the triangular lattice and "spindling" operations. A lower-variance variant: improve the current record for the *fractional* chromatic number of the plane, which is a linear programming problem and much friendlier to automated construction search.

2. Packing 11 unit squares in a square. Let $s(n)$ be the side of the smallest square containing n non-overlapping unit squares. The smallest unresolved case is $n = 11$: the best known packing has side ≈ 3.877, while the best proved lower bound is $2+2\sqrt{4/5} \approx 3.789$. Stromquist proved that

$n = 11$ is the first case where an optimal packing must use angles other than $0°$ and $45°$, which is exactly why the case resists: the rotational degrees of freedom make the branch-and-bound tree explode. Both directions are AI-shaped: evolutionary search for a better packing, and generation plus pruning of the case tree for a certified lower bound via interval arithmetic. An easier warm-up with the same technology: the first unproved case of the densest packing of n congruent circles in a square.

3. Dürer's unfolding conjecture. Every convex polytope can be cut along a spanning tree of its edges and unfolded into the plane without overlap (Shephard, 1975, after Dürer's 1525 treatise). Open. Checking a candidate counterexample is finite: enumerate spanning trees of the edge graph, unfold, test for overlap; and one can define a differentiable "overlap margin" and optimise over shapes. One caveat must be known before starting: O'Rourke proved that *every* combinatorial type of convex polyhedron admits a realisation with an overlapping edge unfolding, so any attack must optimise over metric realisations of a fixed combinatorial type, not over combinatorial types. A better-scoped sub-problem: does every *prismatoid* (the convex hull of two convex polygons in parallel planes) admit an edge-unfolding? This is open, and the shape space is low-dimensional.

4. The Kneser–Poulsen conjecture in $\mathbb{R}^3$. If the centres of finitely many balls are moved so that all pairwise distances weakly decrease, then the volume of their union does not increase. Proved in the plane by Bezdek and Connelly; open in dimension 3 and above. A clean, elementary, believed-true inequality. Here the AI contribution is less about search and more about systematic exploration: test the statement numerically on large random configurations to look for a counterexample (unlikely but decisive if found), and attempt to automate the case analysis in special configurations (few balls, equal radii, symmetric arrangements), where a complete proof of a special case is already a result.

5. The kissing number in dimension 5. How many unit balls can touch a given unit ball in $\mathbb{R}^5$? The answer is known to lie between 40 and 44. The statement needs no background, and both sides are computational: Delsarte-type semidefinite programming for the upper bound (a numerical optimisation over polynomial coefficients, with a rational certificate needed at the end), and configuration search for the lower bound. Closing the gap completely is very hard; improving one of the two bounds by one unit, with a certificate, is not obviously out of reach.

6. Meissner's conjecture (the Blaschke–Lebesgue problem in $\mathbb{R}^3$). Among convex bodies of constant width d in $\mathbb{R}^3$, do the Meissner tetrahedra minimise volume? In the plane the minimiser is the Reuleaux triangle; the three-dimensional statement was conjectured by Bonnesen and Fenchel in 1934 and is open. What makes it attractive as a project is that there is an explicit numerical gap to watch: the best lower bound has moved from Chakerian's $\approx 0.3649\,d^3$ to $(4\pi/33)d^3 \approx 0.3808\,d^3$, against a conjectured minimum $\approx 0.4199\,d^3$. Three possible entry points: push the analytic lower bound, run certified shape optimisation over Meissner polyhedra, or work on reducing the problem to finitely many combinatorial types.

7. Ulam's packing conjecture. Is the ball the worst-packing convex body in $\mathbb{R}^3$? That is, does every convex body admit a packing of density at least that of the densest sphere packing? Open. This is the three-dimensional analogue of Reinhardt's smoothed-octagon problem in the plane (also open). It is attractive precisely because a *counterexample* is plausible: it would be a specific convex body, found by numerical optimisation over a parametrised family of bodies and lattice packings, and then certified.

8. Existence of Butson matrices. A *Butson Hadamard matrix* $\mathrm{BH}(n, q)$ is an $n \times n$ complex matrix whose entries are q-th roots of unity and whose rows are pairwise orthogonal, i.e. $HH^* = nI_n$. For $q = 2$ these are the real Hadamard matrices, and the Hadamard conjecture ($\mathrm{BH}(4k, 2)$ exists for every k) is the most famous special case; the Fourier matrix gives $\mathrm{BH}(n, n)$ for every n; and the Kronecker product of a $\mathrm{BH}(n_1, q_1)$ and a $\mathrm{BH}(n_2, q_2)$ is a $\mathrm{BH}(n_1n_2, \mathrm{lcm}(q_1, q_2))$, so matrices of composite order are abundant while very little is known in prime orders. The general question — *for which pairs* (n, q) *does a* $\mathrm{BH}(n, q)$ *exist?* — is wide open, and current knowledge consists of sporadic constructions together with a handful of non-existence criteria. The basic necessary conditions come from the arithmetic of vanishing sums of q-th roots of unity (results of Lam and Leung and their refinements), which force n to lie in the numerical semigroup generated by the prime divisors of q; further criteria come from algebraic number theory and, very recently, from positive definite functions on finite groups.

This problem is arguably the best-shaped one on the list for an AI-assisted project, for four reasons. (i) The search space is explicitly finite: an $n \times n$ array of elements of $\mathbb{Z}_q$. (ii) Verification is trivial and independent: checking $HH^* = nI_n$ is a few lines of exact arithmetic, so a positive answer is a certificate that nobody has to trust the model for. (iii) The symmetry group (row and column permutations, and multiplication of rows and columns by roots of unity) is large and explicit, so symmetry breaking — the main art in SAT and constraint encodings — is a concrete, well-posed sub-task. (iv) There is a public scoreboard: the online catalogue of Butson matrices of small order maintained at Aalto University[1] has explicit gaps, and filling one of them is a self-contained, publishable result. Recent work illustrates exactly the intended workflow: a new necessary condition was used both to rule out families of pairs (n, q) and to constrain a computer search, which then produced a $\mathrm{BH}(18, 14)$ for the first time, while cyclotomic cosets yielded first examples of $\mathrm{BH}(34, 10)$, $\mathrm{BH}(62, 6)$, $\mathrm{BH}(82, 6)$ and $\mathrm{BH}(146, 6)$ with a 2-circulant structure. Note the structural hypothesis in that last construction: restricting the search to matrices with a prescribed symmetry (circulant, block-circulant, group invariant) is what makes the search space small enough, and choosing a good restriction is a mathematical decision, not a computational one. Concrete targets: fill a gap in the small-order catalogue; or, on the non-existence side, automate the existing criteria and determine how large a region of the (n, q) table they settle.

[1] `https://wiki.aalto.fi/display/Butson/Matrices+up+to+monomial+equivalence`