

Arithmetic Billiards

Arithmetic billiards are a geometric construction which allow an interesting visualisation for the greatest common divisor and the least common multiple of two positive integers. For an introduction to the subject, see [this article](#).

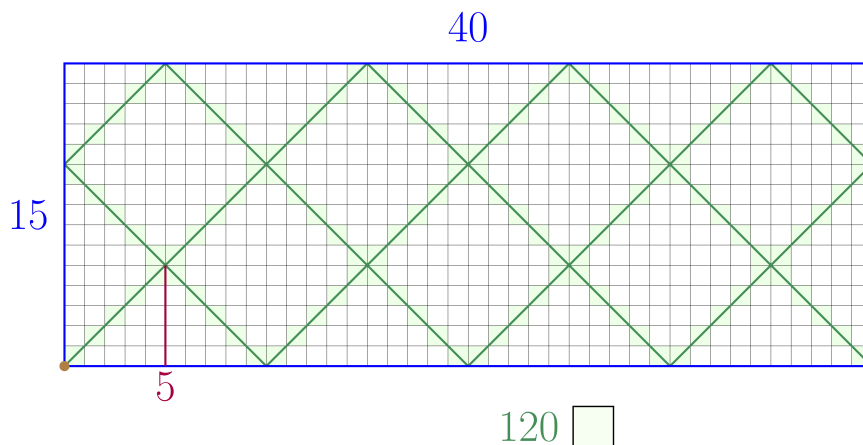


Figure 1: A billiard table with sides 15 and 40. The first point of self-intersection of the path is at distance $5\sqrt{2} = \gcd(15, 40)\sqrt{2}$ from the corner and the total length of the path is $120\sqrt{2} = \text{lcm}(15, 40)\sqrt{2}$.

Goal of the project

In this project we would like to explore some generalizations of arithmetic billiards, namely the n -dimensional case – where the rectangular table is replaced by an n -dimensional parallelepiped – or different shapes in the two-dimensional case – for example an L-shaped table or a square table with a square “hole” in the middle.

In each case our aim is studying the path of the ball and in particular some of its “special” points, such as the points where the ball bounces on the sides of the table or those where the path self-intersects. More precisely, we would like to count these points and describe their coordinates and multiplicity (the number of times the path goes through that point) and relate these quantities to the greatest common divisor and least common multiple of the length of the sides.

In order to do so, the student has to write a small program that, given as input the size of the billiard table, computes the desired information about the special points mentioned above. Any programming language is fine for this part.

