Math for Labs: Balancing Centrifuges

LEVEL: EML2, EML4, Student Project, Bachelor/Master Thesis

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THE CLASSICAL PROBLEM

The main aim of a lab centrifuge is separating fluids that have distinct densities. It has holes meant to contain test tubes, and these holes are regularly arranged in a circle around the rotation center. How many holes? That's the question. Indeed, the test tubes have to be placed in a kind of symmetric way so that the centrifuge is balanced. Notice that a centrifuge could be permanently damaged because of unbalanced test tubes.

In this problem there are two parameters: n, the numbers of holes of the centrifuge; k, the number of test tubes that one would like to balance. One can determine (for small numbers) the pairs n, k for which this is possible, and the actual configurations of the test tubes that achieve balance.
The mathematical model for this problem is a set of roots of unity (more precisely, k roots of unity of order dividing n) in the complex plane that add to zero.

NATURAL DIRECTIONS OF EXPLORATIONS

The classical problem has been solved by Sivek, and the first part of the project aims at understanding his short and relatively easy proof (3 pages). Then we aim at investigating some of the following variants, that may still be (in part) open.

- The classical problem asks about k samples in n holes arranged regularly on a circle. What about more circles? So can one build a centrifuge with two (possibly, three) circles so that it is balanced for many values of k? And what would be the optimal amount of holes in each circle?

- Are there many (more precisely, how many) configurations that balance k samples in n holes? Which are the values of k maximizing the amount of possibilities?
- What if one allows the samples to contain less liquid than others? This would be like having a ”weighed centrifuge”, where each sample has a weight. In some sense, this is like ”repeating samples”, so one reference could be the work of T. Y. Lam and K. H. Leung.
- A related problem, mentioned in M. Baker's celebrated blog entry on the subject, is called *the hanging baskets problem*. A countinuous version of the problem seems to have been considered by G. Zangerl and A. Steinicke.
- After solving the mathematical problems, let's come to ”practical problems”. Looking at the pictures of the centrifugues it is pretty easy to make mistakes as what goes where. Would there be a way of coloring or coding the labels on the centrifuge to make mistakes less likely?

The above mathematical questions can be answered with the help of a computer for small numbers (bounding the numbers makes sense for the practical problem) but studying the general questions mathematically is also possible and surely very interesting.

The output of this project will be material (in a broad sense) that can be used for outreach activities (in a broad sense). In the very best case (of course, one can never promise a mathematical paper in advance) we will write a small article in recreational mathematics. Motivated students at all levels are welcome to join!

References

T. Y. Lam and K. H. Leung (1996)
https://arxiv.org/pdf/math/9605216.pdf

G. Sivek (2010)
http://www.kurims.kyoto-u.ac.jp/EMIS/journals/INTEGERS/papers/k31/k31.pdf

M. Baker (2018)
https://mattbaker.blog/2018/06/25/the-balanced-centrifuge-problem/

G. Zangerl and A. Steinicke (2022)
https://dergipark.org.tr/en/download/article-file/1885012