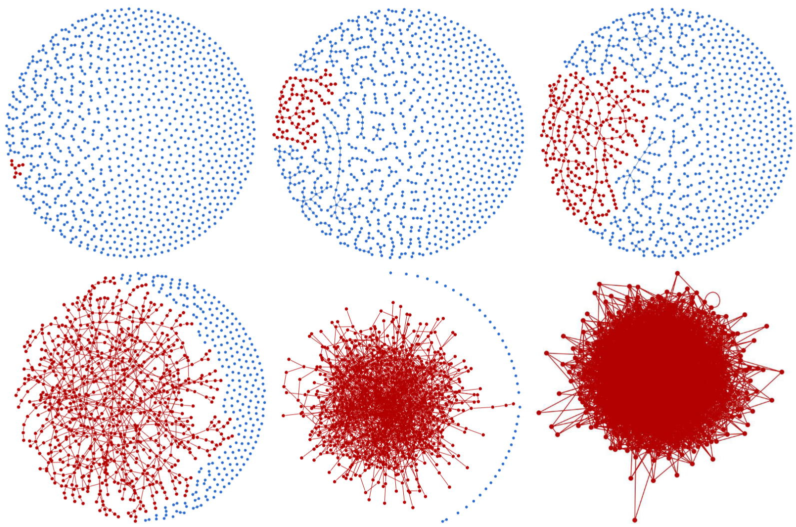
Sharp transitions for percolation in Erdős-Rényi graphs

Experimental Mathematics Lab project



(Simulation by Nicolas Curien)

Abstract. Consider a graph with n (labelled) vertices and M edges, uniformly chosen among all such graphs. As in the picture above, consider the connected components of it, and order them according to size. For $M \simeq n/2$ (top middle), one can see on simulations that the biggest component is actually fairly small; for instance, the probability that it makes up at least 1% of the graph vanishes for n very large. On the other hand, for $M \simeq 2$ (bottom left), a macroscopic component appears; actually, at least 20% of the vertices will belong to a single cluster with large probability.

Actually, it is part of a constellation of results of Erdős and Rényi that the transition is in fact sharp: for all $c < 1$ and $M \leq cn$, we are in the first situation, where large components do not appear; for all $c > 1$, we are in the latter, where the largest component makes up a positive proportion of the graph. See https://youtu.be/mpe44sTSoF8 for a simulation of the above phenomenon, where the transition occurs where the parameter is roughly 1.

The first objective of the project is to simulate such large random graphs and their connected components. Later, we can study different notions of small- and largeness for those, and check experimentally if one should expect a transition, and whether it seems to be sharp or not.

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