Printable planetarium

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Language: French or English

Description: The aim of this project is to create with a 3D printer a planetarium representing the rotations of the earth and an other planet around the Sun.

Let us consider the planet Saturn. It takes approximatively $\frac{77708431}{2640858} = 29,425448...$ years for Saturn to make a revolution around the Sun. This means that if Saturn completes 2640858 rotations around the Sun, the earth must complete 77708431 rotations. This could be modelled as in Figure 1 with three wheels, the one of Saturn having 77708431 teeth and the one of the Earth having 2640858 teeth. Of course, it would be completely impossible to print such wheels with so much teeths and we need to find a fraction with smallest denominator and numerator but that is still well approximating the period of revolution for Saturn.

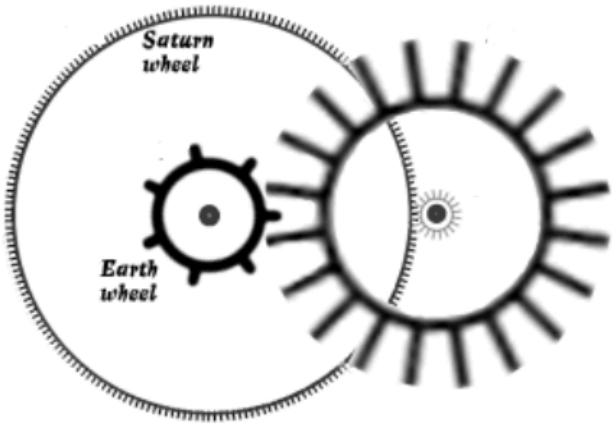

Figure 1: Wheels representing the motions of planets Earth and Saturn

Continued fraction is the representation of a real number r in the form

$$r = a_0 + \cfrac{1}{a1+\cfrac{1}{a_2+\cfrac{1}{a_3+\cfrac{1}{\ddots}}}}.$$

In this case, we write $r = [a_0; a_1\, a_2\, a_3 \cdots]$. If $r \in \mathbb{R} \setminus \mathbb{Q}$ this writing is unique and the expansion is infinite while, for a rational number, there are two different ways to represent it as a finite continued fraction.

One of the most interesting features of continued fractions is the fact that if

$r = [a_0; a_1\, a_2\, a_3 \cdots]$ and $\frac{p_k}{q_k}$ is the rational number $[a_0; a_1, \cdots, a_k]$ then

$$\frac{1}{q_k(q_{k+1} + q_k)} < \left| r - \frac{p_k}{q_k} \right| < \frac{1}{q_k q_{k+1}}.$$

Moreover, this approximation is the best, in a way that the students would have to understand.

The aim of this project is to study and understand the first basic notions of continued fractions in order to create algorithms which are able to decompose a real number into finite continued fractions with a certain degree of precision. This algorithm will be applied to find better wheels to build the planetarium.

This project is inspired by Huygens planetarium.