Areas of convex hulls and Viterbo's conjecture

EML project, winter semester 2023

Branch: Geometry
Language: English
Difficulty: adjustable
Trailer: https://www.youtube.com/watch?v=5gXBIww-wxA
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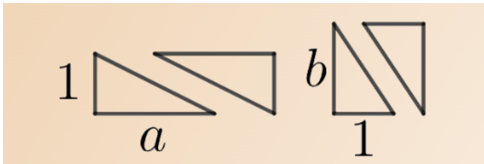
Teaser

Someone gives you a few 2D shapes on the plane, for example:

(Easy) Two regular triangles, one being the 180° rotated copy of the other.

(Hard) Four triangles as in the picture on the right.



You are allowed to parallel translate the shapes in your set (move them around without rotating them), and the goal is to minimize the area of the convex hull [1]. What is the minimal area? When is it attained?

Project goal

We'll consider a few instances of this problem (determined by the set of shapes one starts with) and try to play with them in Geogebra and Sage to figure out answers experimentally.

Motivation

This problem is very cool because it allows to solve special cases of *Viterbo's conjecture*! This is something fancy, related to *symplectic geometry*, and the reduction of the conjecture to this problem is done using a very neat *billiard* approach [2].

References

[1] https://en.wikipedia.org/wiki/Convex_hull.

[2] Balitskiy, A. "Equality cases in Viterbo's conjecture and isoperimetric billiard inequalities." International Mathematics Research Notices 2020.7 (2020): 1957-1978. Arxiv version: https://arxiv.org/abs/1512.01657.