# Magic Squares

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# 1 Introduction

Magic squares have been used for centuries and this all around the world. They have been found in ancient China, where they were called *Lo-Shu*, and were linked to religious purposes as a key to communicate with the gods. Magic squares were also considered to be a symbol that would reunite all the basic principles of the universe. Magic squares were also found early in India and were later on brought to the Western culture by the Arabs. Some of the squares used by the Arabs were later used in conjunction with magic letters so as to assist Arab illusionists and magicians. During the Middle Ages, the importance of magic squares grew, since they were then used in astrology, horoscopes and talismans, as magic squares were said to have special and mysterious features that would reveal many hidden secrets.

In this report, the main focus will, on one hand, be on the general definitions, properties and examples of magic squares, and, on the other hand, their construction, including the construction on *Sage*, in order to visualise some magic squares using different construction methods. The parameters used in the *Sage*-document attached will also be explained.

# 2 Definitions and properties

## 2.1 Definition and properties of a magic square

In this report, we will developpe the construction of a (simple) magic square which we define as : an arrangement of different numbers, placed in a square grid by following a certain pattern, so that it has the same sum for any row, any column or any main digonal. This sum is then called the magic constant of this magic square.

The number of rows and columns will be $n$ for the purpose of this essay with $n \in \mathbb{N}$.

The numbers are usually integers and in many cases the first number to be placed in the square is 1. Each number is only used once. A magic square therefore always has $n^2$ numbers. A magic square with the integers from 1 to $n^2$ in arithmetic progression with common difference of 1 is called a normal magic square.

There are many types of magic squares. All magic squares have the same basic properties, but some magic squares have some additional properties that make them more interesting. Every magic square with the basic properties has a so called magic constant, which is the sum of every row or every column or every main diagonal. In fact, a magic square has the same sum for any row, any column or any main diagonal. This magic constant can be calculated with the formula:

$$M = \frac{n(n^2 + 1)}{2}$$

Any magic square maintains its properties, even if it is reflected or rotated. By doing so, 8 trivially different squares are produced. In magic square theory all of these are generally deemed equivalent and the eight such squares are said to make up a single equivalent class.

According to D.N.Lehmer, a magic square is an arrangement such that it has the same sum, which is equal to the magic constant for any row or any column, but the sum of the main diagonals doesn't matter here. We will use this definition for the construction of magic square (based on D.N.Lehmer definition) using the uniform step method. Later, we will see that D.N.Lehmer's definition correspond to the definition of a semi-magic square.

**Example 1:**

| | | | | |
|---|---|---|---|---|
| 17 | 24 | 1 | 8 | 15 |
| 23 | 5 | 7 | 14 | 16 |
| 4 | 6 | 13 | 20 | 22 |
| 10 | 12 | 19 | 21 | 3 |
| 11 | 18 | 25 | 2 | 9 |

Figure 1 – A normal magic square of order 5

We observe that the magic constant of a magic square of order 5 is 65:

— $17 + 24 + 1 + 8 + 15 = 65$ and this will be the same for any other row

— $17+23+4+10+11 = 65$ and this will be the same for any other column

— $17 + 5 + 13 + 21 + 9 = 65 = 15 + 14 + 13 + 12 + 11$

— $\frac{5(25+1)}{2} = 65$

| | | | | |
|---|---|---|---|---|
| 17 | 24 | 1 | 8 | 15 |
| 23 | 5 | 7 | 14 | 16 |
| 4 | 6 | 13 | 20 | 22 |
| 10 | 12 | 19 | 21 | 3 |
| 11 | 18 | 25 | 2 | 9 |

| | | | | |
|---|---|---|---|---|
| 11 | 10 | 4 | 23 | 17 |
| 18 | 12 | 6 | 5 | 24 |
| 25 | 19 | 13 | 7 | 1 |
| 2 | 21 | 20 | 14 | 8 |
| 9 | 3 | 22 | 16 | 15 |

| | | | | |
|---|---|---|---|---|
| 15 | 8 | 1 | 24 | 17 |
| 16 | 14 | 7 | 5 | 23 |
| 22 | 20 | 13 | 6 | 4 |
| 3 | 21 | 19 | 12 | 10 |
| 9 | 2 | 25 | 18 | 11 |

Figure 2 – Three magic squares of order 5

These three magic squares of order 5 have the same properties, since they are the rotation and the reflection of the same magic square.

**Example 2:**

Taking a magic square of order 3, there are 8 equivalent magic squares.

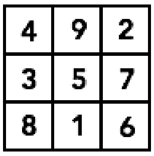



Figure 3 – A magic square of order 3

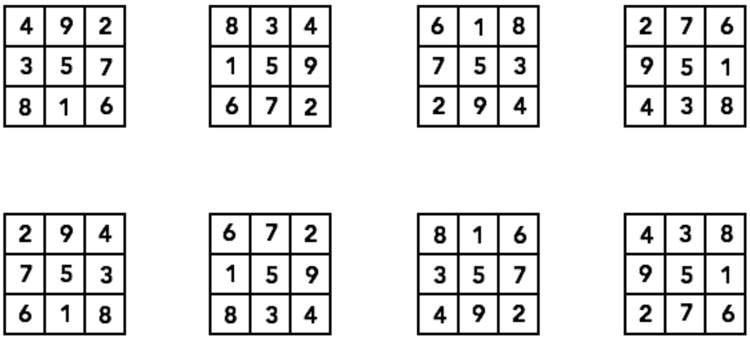
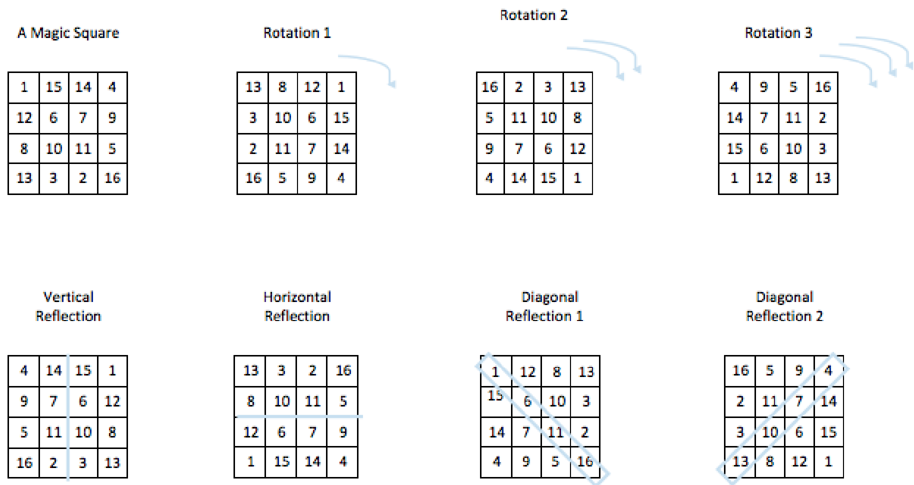



Figure 4 – The rotations and reflections of a magic square of order 3

## Example 3:

In this example, it is possible to see how those rotations and reflections happen.




## <u>Functions used in *Sage*</u>

The following functions were used for the construction and visualisation of the rotations and reflections of magic squares in *Sage*:

**EquivSR :** The function that takes a magic square MS and its order in arguments and returns the magic square SR, which is equal to the rotation of MS.

**EquivDR :** The function that takes a magic square MS and its order in arguments and returns the magic square DR, which is equal to the double rotation of MS.

**EquivTR :** The function that takes a magic square MS and its order in arguments and returns the magic square TR, which is equal to the triple rotation of MS.

**EquivDP :** The function that takes a magic square MS and its order in arguments and returns the magic square DP, which is equal to the reflection of MS with respect to the first diagonal.

**EquivDS :** The function that takes a magic square MS and its order in arguments and returns the magic square DS, which is equal to the reflection of MS with respect to the second diagonal.

**EquivAV :** The function that takes a magic square MS and its order in arguments and returns the magic square AV, which is equal to the reflection of MS with respect to the vertical line centered in the middle of the columns of MS.

**EquivAH :** The function that takes a magic square MS and its order in arguments and returns the magic square AH, which is equal to the reflection of MS with respect to the horizontal line centered in the middle of the rows of MS.

**EquivMat :** The function that takes a magic square MS and it's order in arguments and returns a list [MS, SR, DR ,TR, DP, DS, AV, AH] of magic squares, the equivalent class of MS, e.g; all the symmetries of MS.

## 2.2 Odd, even and double even magic squares

Depending on the parity of the order of the magic square, it is possible to do a first classification of these squares: odd, even and double even magic squares.

### Odd magic squares

When the order of the magic square is odd, it is called an odd magic square.

| | | |
|---|---|---|
| 8 | 1 | 6 |
| 3 | 5 | 7 |
| 4 | 9 | 2 |

Figure 5 – An odd magic square of order 3

Its magic constant is 15.

### Single even magic squares

Just as for odd magic squares, a single even magic square is a magic square of order $n$, with $n$ being pair, but not a multiple of 4 ; so if its order is such that $n = 4k + 2$ with $k \in \mathbb{N}$.

| | | | | | |
|---|---|---|---|---|---|
| 35 | 1 | 6 | 26 | 19 | 24 |
| 3 | 32 | 7 | 21 | 23 | 25 |
| 31 | 9 | 2 | 22 | 27 | 20 |
| 8 | 28 | 33 | 17 | 10 | 15 |
| 30 | 5 | 34 | 12 | 14 | 16 |
| 4 | 36 | 29 | 13 | 18 | 11 |

Figure 6 – An even magic square of order 6

Its magic constant is 111.

Remark: There is no magic square of order 2. The smallest double even magic square is of order 4 and the smallest single even magic square is of order 6.

### Double even magic squares

A magic square is double even if its order n is a multiple of 4, so if its order is such that $n = 4k$ with $k \in \mathbb{N}$.

| | | | |
|---|---|---|---|
| 2 | 7 | 12 | 13 |
| 16 | 9 | 6 | 3 |
| 5 | 4 | 15 | 10 |
| 11 | 14 | 1 | 8 |

Figure 7 – A double even magic square of order 4

Its magic constant is 34.

# 3 Different types of magic squares

Although magic squares have the same basic properties, some have additional properties, making them even more intriguing than the normal ones. Others have the usual properties, but fail in one or two. Magic squares can therefore also be classified depending on the additional or leaking properties they have in comparison with the basic properties of a magic square.

For this section, every time a colour is used in a magic square, the colour represents the numbers that add up to a certain sum. By using different colours in the same magic square, we show that the sum of the numbers of one colour is the same as the sum of the numbers of the other colour.

## 3.1 Semimagic squares

A semimagic square is a magic square of order $n$ that has all the properties of a normal magic square, but fails when it comes to the sum of the main diagonals. In fact, a magic square is called a semimagic square, if the sum of one or two of the main diagonals is not equal to the magic constant.

| | | | |
|---|---|---|---|
| 1 | 2 | 15 | 16 |
| 6 | 11 | 7 | 10 |
| 13 | 12 | 4 | 5 |
| 14 | 9 | 8 | 3 |

Figure 8 – Example of a semimagic square of order 4

In this example, the sum of each row and column is 34, but the sum of the numbers of the main diagonal $(1, 11, 4, 3)$ is 19 and the sum of the numbers of the other main diagonal $(16, 7, 12, 14)$ is 49.

## 3.2 Associative magic squares

An associative magic square is a magic square of order $n$ for which every pair of numbers symmetrically opposite to the center sum to $n^2 + 1$.

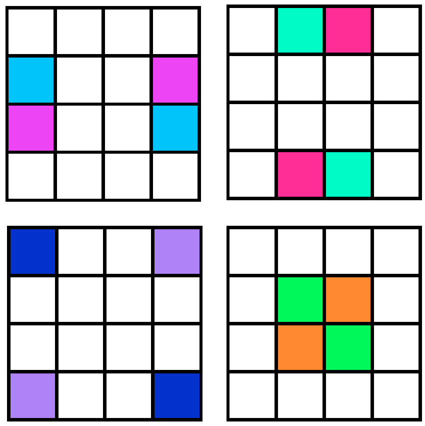


Figure 9 – The numbers which have the same sum

| | | | |
|---|---|---|---|
| 2 | 16 | 13 | 3 |
| 7 | 9 | 12 | 6 |
| 11 | 5 | 8 | 10 |
| 14 | 4 | 1 | 15 |

Figure 10 – Example of an associative magic square of order 4

| | | | |
|---|---|---|---|
| 2 | 16 | 13 | 3 |
| 7 | 9 | 12 | 6 |
| 11 | 5 | 8 | 10 |
| 14 | 4 | 1 | 15 |

This example is an associative magic square of order 4 and therefore the sum of every pair of numbers symmetrically opposite to the center is 17 since $4^2 + 1 = 17$.

## 3.3 Multimagic squares

### Bimagic squares

A magic square is called bimagic, or 2-multimagic, if it remains magic after each of its numbers has been squared.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 22 | 3 | 81 | 42 | 34 | 47 | 17 | 59 | 64 |
| 37 | 54 | 15 | 71 | 76 | 57 | 32 | 20 | 7 |
| 33 | 38 | 8 | 55 | 72 | 77 | 52 | 13 | 21 |
| 68 | 73 | 43 | 12 | 26 | 4 | 63 | 51 | 29 |
| 2 | 16 | 58 | 46 | 41 | 36 | 24 | 66 | 80 |
| 53 | 31 | 19 | 78 | 56 | 70 | 39 | 9 | 14 |
| 61 | 69 | 30 | 5 | 10 | 27 | 74 | 44 | 49 |
| 75 | 62 | 50 | 25 | 6 | 11 | 67 | 28 | 45 |
| 18 | 23 | 65 | 35 | 48 | 40 | 1 | 79 | 60 |

Figure 11 – Example of a bimagic square

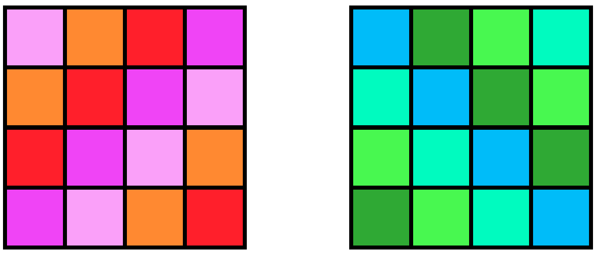
In this example, if we elevate each number to the power of 2 we obtain another magic square.It is therefore bimagic.

#### Multimagic squares

The definition of a bimagic square makes it easy to understand what a multimagic square is. It's just like a bimagic square, a magic square in which every number has been elevated to a certain power producing a new magic square. A square is $P$-multimagic if it remains magic after each of its numbers has been replaced by their $k$-th power (for $k = 1, 2, ..., P$)

### 3.4 Panmagic/Diabolic magic squares

Panmagic squares, also known as diabolic squares due to the difficulty in their construction, are magic squares with the additional property that all the diagonals sum to the same magic constant. This also means that, considering a magic square as a vertical and horizontal cylinder, every diagonal obtained by "wrapping around" the edges of the square also sums to the same magic constant.



| | | | |
|---|---|---|---|
| 1 | 8 | 10 | 15 |
| 12 | 13 | 3 | 6 |
| 7 | 2 | 16 | 9 |
| 14 | 11 | 5 | 4 |

Figure 12 – Example of a panmagic square of order 4

In fact, by taking one of the diagonals formed by 'wrapping around' and summing the numbers, the magic constant 34 is obtained.

| | | | |
|---|---|---|---|
| 1 | 8 | 10 | 15 |
| 12 | 13 | 3 | 6 |
| 7 | 2 | 16 | 9 |
| 14 | 11 | 5 | 4 |

$12 + 2 + 5 + 15 = 34$

### 3.5 Heterosquares

The big difference between normal magic squares and heterosquares of order $n$ is that the latter doesn't respect the main property that each row, each column and each principal diagonal have the same sum. A heterosquare will have a different sum.

| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 5 | 6 | 7 | 8 |
| 9 | 10 | 11 | 12 |
| 13 | 14 | 16 | 15 |

Figure 13 – Example of a heterosquare of order 4

A special case of heterosquares is the antimagic square, in which these different sums form a sequence of consecutive integers.

| | | | |
|---|---|---|---|
| 15 | 2 | 12 | 4 |
| 1 | 14 | 10 | 5 |
| 8 | 9 | 3 | 16 |
| 11 | 13 | 6 | 7 |

Figure 14 – Example of an antimagic square of order 4

In this example:

— Sums of the rows

First row: **33**
Second row: **30**
Third row: **36**
Fourth row: **37**

— Sums of the columns

First column: **35**
Second column: **38**
Third column: **31**
Fourth column: **32**

— Sums of the principal diagonals

Principal diagonal 15, 14, 3, 7: **39**
Principal diagonal 4, 10, 9, 11: **34**

The sums of the rows, columns and main diagonals form a sequence of consecutive integers: $30, 31, 32, 33, 34, 35, 36, 37, 38, 39$.

# 4 Famous magic squares

As mentioned above, magic squares have been been there for many centuries. Magic squares have intrigued many people, from painters to scientists. In this section, two famous magic squares will be presented: one constructed by Benjamin Franklin and another one by Albrecht Dürer.

## 4.1 Benjamin Franklin's magic square

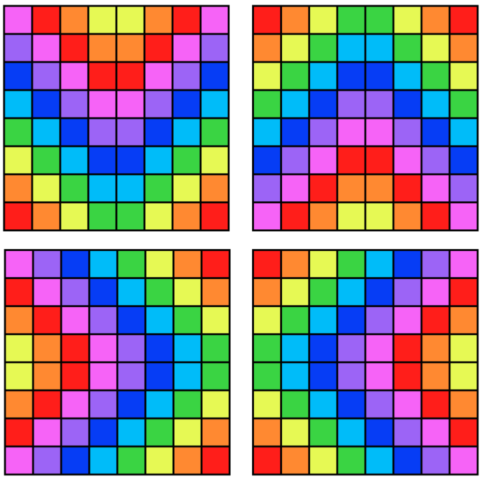
Benjamin Franklin created two well known magic squares, one of order 8 and another of order 16. For the purpose of this essay, only the properties of the magic square of order 8 which was created in 1750 will be shown.

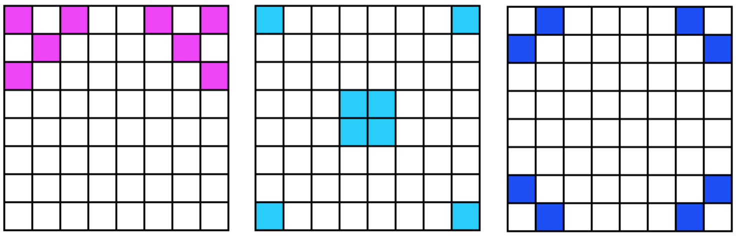
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 52 | 61 | 4 | 13 | 20 | 29 | 36 | 45 |
| 14 | 3 | 62 | 51 | 46 | 35 | 30 | 19 |
| 53 | 60 | 5 | 12 | 21 | 28 | 37 | 44 |
| 11 | 6 | 59 | 54 | 43 | 38 | 27 | 22 |
| 55 | 58 | 7 | 10 | 23 | 26 | 39 | 42 |
| 9 | 8 | 57 | 56 | 41 | 40 | 25 | 24 |
| 50 | 63 | 2 | 15 | 18 | 31 | 34 | 47 |
| 16 | 1 | 64 | 49 | 48 | 33 | 32 | 17 |

Figure 15 – Benjamin Franklin's magic square of order 8

Even though this magic square is a semimagic square, it has some additional properties. Here, the sum of every small square with the same colour has the magic constant 260.

Franklin's magic square has other combinations of numbers with the same sum:





Besides the properties mentioned above, two others can be mentioned. The first one: every four entries in every $2 \times 2$ subsquare sum to 130; the second one: all half row and half columns also sum to 130.

## 4.2 Albrecht Dürer's magic square

Albrecht Dürer was a German painter born in 1471 and dead in 1528. In 1514 he introduced a very interesting magic square in one of his paintings . In *Melencolia I*, Dürer inserted a magic square of order 4 with some interesting properties, showing some additional properties, such as the numbers 15 and 14 that are in the center of the last row, which give the year where *Melencolia I* was released: 1514.

| | | | |
|---|---|---|---|
| 16 | 3 | 2 | 13 |
| 5 | 10 | 11 | 8 |
| 9 | 6 | 7 | 12 |
| 4 | 15 | 14 | 1 |

Figure 16 – Albrecht Dürer's magic square in *M*elencolia I

As seen before, the magic constant of a normal magic square of order 4 is 34. But this magic constant doesn't only appear in the sum of the rows, columns and principal diagonals from Dürer's magic square. In fact, it is possible to see that 34 appears in many other sums:

| | | | |
|---|---|---|---|
| 16 | 3 | 2 | 13 |
| 5 | 10 | 11 | 8 |
| 9 | 6 | 7 | 12 |
| 4 | 15 | 14 | 1 |

Figure 17 – The sum of the numbers of each quadrant is 34

| | | | |
|---|---|---|---|
| 16 | 3 | 2 | 13 |
| 5 | 10 | 11 | 8 |
| 9 | 6 | 7 | 12 |
| 4 | 15 | 14 | 1 |

Figure 18 – The sum of the 4 numbers in the center of the square is 34

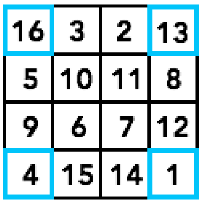



Figure 19 – The sum of the numbers of the 4 corners is 34

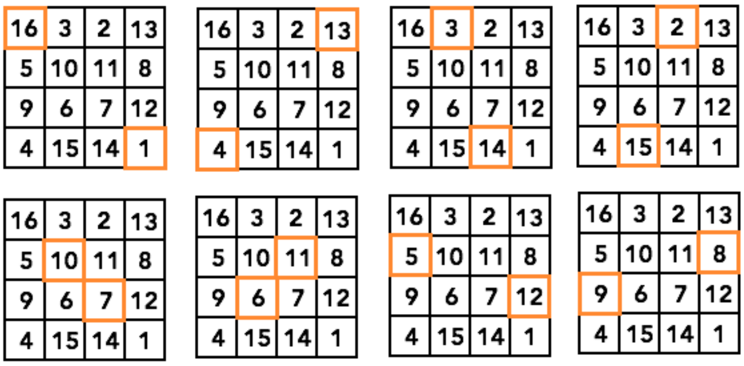



Figure 20 – The sum of every pair of numbers symmetrically opposite to the center is 17

# 5 Construction and Visualisation of magic squares

There are many methods to construct magic squares. Here, the focus will be:

1. on the *Strachey Method* for the construction of even magic squares;
2. on the *Diagonal Method* for the construction of double even magic squares;
3. on the *Siamese Method* for the construction of odd magic squares;
4. on the *Uniform Step Method* for the construction of magic squares, according to D.N.Lehmer's definition.

For this report *Sage* was used for the construction and visualisation of magic squares of order $n$. This section aims to explain the different methods of construction and the parameters used in the *Sage Code* which can be found in the attached document.

For any number k in the magic square of order $n$ we can write $M_{i,j} = k$. Then the number k placed in the $(i+1)^{th}$ row of the $(j+1)^{th}$ column of our square. We have $0 \leqslant i \leqslant (n-1)$ and $0 \leqslant j \leqslant (n-1)$ with $i = i[n]$ and $j = j[n]$.

Visualisation :

For this visualisation of magic squares, the following function was used in *Sage*:

**Visualisation :** Taking a matrix square in parameter, this function defines the visualisation of this matrix. Return a magic square if and only if the number in the matrix are in a specific arrangement.

## 5.1 Odd magic squares

### 5.1.1 Siamese method

This method is based on the following criteria:

— The number 1 is placed in the middle of the first row, hence $M_{1,\frac{n-1}{2}} = 1$.

— From there, for every $x$ in $[2, n^2]$, with $M_{i,j} = x - 1$ then :
If $1 = x[n]$ ; then $M_{i+1,j} = x$.
If not ; then $M_{i-1,j+1} = x$

For this first method, the following functions were used in *Sage*:

**f(n) :** $f(n)$ is the function taking an odd $n$ in parameter and which returns a matrix M of order $n$ with only 0 in every placement except 1 in the middle of the first row.

**MSConstructOddSM :** The function, taking an odd n as parameter and which return a matrix of order $n$ with the numbers from 1 to $n^2$

in the correct placement to be a magic square, but, instead of being in the square grid, it's in a matrix. The first criteria mentioned above are used to generate the right placements of the numbers.

#### 5.1.2 Uniform step method

This method is used to creates an odd magic square according to D.N.Lehmer : a magic square is an arrangement such that it has the same sum, which is equal to the magic constant for any row or any column.

For every $x$ in $(1, 2, ..., n^2)$, we denote $M_{A_x,B_x} = x$ the coordinates of $x$ in a magic square of order $n$ with $A_x = A_x[n]$ and $B_x = B_x[n]$.

To simplify the reading of those coordinates, we will also use the following notation : $M(A_x, B_x) = x$.

To create an magic square, we can take 6 integers $i, j, \alpha, \beta, a$ and $b$ such that $i$ and $j$ are randomly chosed in $[0, n-1]$ and we will explain bellow how to choose the left four integers. Then we set :
$M(i, j) = 1$ ; $M(i + \alpha, j + \beta) = 2$ ; $M(i + 2\alpha, j + 2\beta) = 3$
This goes on until number $n$.

A break-step is now needed, otherwise the following situation would appear: The number $n + 1$ would be placed in the cell $(i + n\alpha, j + n\beta)$, which is the same as the cell $(i, j)$ as can be seen with: $i + n\alpha \equiv p \mod (n)$ and $j + n\beta \equiv q \mod (n)$

Break-step: The number $(n + 1)$ is placed in the cell $(i + a, j + b)$. We then continue:
— $M(i + a + \alpha, j + b + \beta) = n + 2$.
— $M(i + a + 2\alpha, j + b + 2\beta = n + 3)$.
This goes on until $2n$.

Break-step: $M(i + 2a, j + 2b) = 2n + 1$
— $M(i + 2a + \alpha, j + 2b + \beta) = 2n + 2$.
And so on...

This process is equivalent to: For every $x \in 1, 2, ..., n^2$, $M_{A_x,B_x} = x$ such that:

$$A_x \equiv i + \alpha(x - 1) + a[\frac{x-1}{n}] \mod (n)$$

$$B_x \equiv j + \beta(x - 1) + b[\frac{x-1}{n}] \mod (n)$$

**Theorem.** *Given $n \in \mathbb{N}$, the square $n \times n$ constructed by the uniform step method with $a$, $b$, $\alpha$ and $\beta$ is magic if and only if: $a$, $b$, $\alpha$ and $\beta$ and* $\det \begin{vmatrix} \alpha & a \\ \beta & b \end{vmatrix}$ *are prime to $n$.*

The following results are also obtained:

— The magic square $\begin{pmatrix} \alpha & a \\ \beta & b \end{pmatrix}$ is <u>diabolic</u> if and only if the numbers $\alpha \pm \beta$ and $a \pm b$ are prime to $n$.

— The magic square $\begin{pmatrix} \alpha & a \\ \beta & b \end{pmatrix}$ is <u>symmetric</u> if and only if $2i \equiv \alpha + a + 1 \mod (n)$ and $2j \equiv \beta + b + 1 \mod (n)$

**<u>Functions used in *Sage*</u>**

For this second method, the following functions were used in *Sage*:

**MSConstructOddUSMWAA :** The name of the function, with parameters $n, i, j, \alpha, \beta, a$ and $b$, , which returns an matrix with the numbers from 1 to $n^2$ in the correct placement to be a magic square, if and only if $i$ and $j$ are in $[1, n]$ $\alpha, \beta, a, b$ and $\alpha b - \beta$ are prime to $n$.

**pgcd :** The name of the function, with arguments $a$ and $b$, $a$ and $b$ being two integers, which returns the greatest common divisor of $a$ and $b$ if and only if $b \leqslant a$.

**det :** The name of the function taking a list of four numbers $(\alpha, \beta, a, b)$ as argument, which returns the number $\alpha b - \beta a$.

**FourPrimeFor :** The name of the function, with argument $n$ odd, which return a list with four integers $(\alpha, a, \beta, b)$ prime to $n$.

**MSConstructOddUSMWNA :** The name of the function, with parameter $n$, which returns a matrix with the numbers from 1 to $n^2$ in the correct placements to be a magic square. This function works only if $n$ is odd, and if det(FourPrimeFor(n)) is prime to $n$.

## 5.2 Even magic squares

For an even magic square M of order $n$ there are two possibilities:

1. M is a single even order magic square if its order is divisible by 2 but not divisible by 4.
2. M is a double even order magic square if its order is divisible by 4.

### 5.2.1 Single even magic squares: Strachey Method

A single even magic square is a magic square whose order is divisible by 2 but not divisible by 4. The method used in this section to construct a $n \times n$ single even magic square will be the *Strachey Method*.

Let $n = 2u$ with $u \in \mathbb{N}$ such that $u$ is odd in order not to be divisible by 2. Now we construct 4 magic squares $(A, B, C, D)$ of order $u$ by following the siamese method but instead of having the number 1 in the middle of the first row, the following numbers are put instead:

— 1 for A such that A contains every number between 1 and $u^2$.

— $u^2 + 1$ for C such that C contains every number between $u^2 + 1$ and $2u^2$.

— $2u^2 + 1$ for B such that B contains every number between $2u^2 + 1$ and $3u^2$.

— $3u^2 + 1$ for D such that D contains every number between $3u^2 + 1$ and $4u^2$.

Let $u = 2k + 1$ with k $\in \mathbb{N}$.
The next steps follow:

— The first $k$ columns in the sub-square A are exchanged with the corresponding columns of sub-square C.

— The last $k - 1$ columns in the sub-square B are exchanged with the corresponding columns of sub-square D.

— The middle cell of the first column of sub-square A is exchanged with the middle cell of the first column of sub-square C.

— The central cell in sub-square A is exchanged with the central cell in sub-square C.

— Finally, the four squares are put with A in the left up corner, B in the right up corner, C in the left down corner and D in the right down corner..

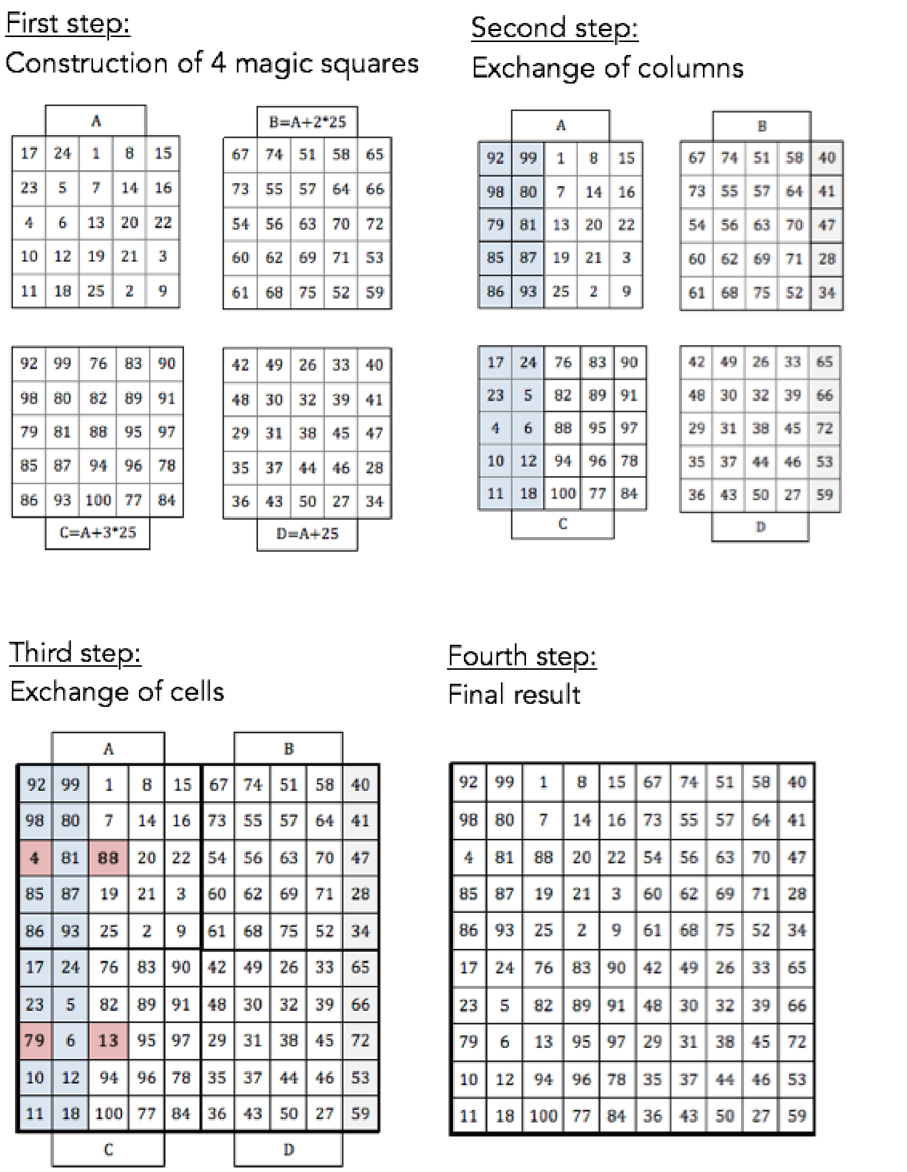



Figure 21 – Example of the construction of a single even magic square of order 10 by the Strachey Method

## Functions used in *Sage*

For the Strachey Method the following functions were used in *Sage*:

**MSConstructSEven :** The name of the function, with parameter $n$, which returns a matrix with the numbers from 1 to $n^2$ in the correct placement to be a magic square, if n is a single even.

**assemblematrices :** The name of the function which takes as argument four matrices (A,B,C,D) and their order $u$, and returns a matrix of order $2u$ with A in the left up corner, B in the right up corner, C in the left down corner and D in the right down corner.

.

### 5.2.2 Double even magic squares: Diagonal method

A double even magic square is a magic square whose order is divisible by 4. The diagonal method will be used for the construction of a $n \times n$ double even magic square.

Let $u = \frac{n}{4}$ be in $\mathbb{N}$ since n is divisible by 4.

— Create a $n \times n$ matrix M filled with increasing numbers in $\mathbb{N}$ from 1 to $n^2$, from the left top to the bottom right corner.

— Separate M in $u^2$ matrices of order 4 and for each of them, delete all numbers that aren't on the diagonals.

— Fill the rest of M from the right bottom to the left top, with increasing numbers in $[1, n^2]$ that aren't already in the matrix M.

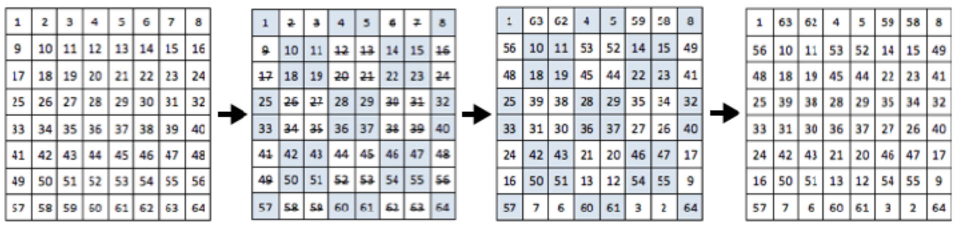


Figure 22 – Example of a double even magic square of order 8 constructed by the Diagonal Method

<u>**Functions used in *Sage***</u>

For this method, the following fonctions were used in *Sage*:

**MSConstructDEven :** The name of the function, with parameter $n$, which returns a matrix with the numbers from 1 to $n^2$ in the correct placement to be a magic square if n is a double even.

## 6 Magic cubes

The principle of magic squares can be extrapolated from two dimensions to any number of higher dimensions. A relevant example is a magic cube that will satisfy properties similar to the properties of a magic square. A magic cube can be compared to a magic square, but, instead of two dimensions, it has three. It contains $n^3$ numbers and all rows, all columns, all pillars and the union of the four main space diagonals, sum to a magic constant that can be calculated by the formula:

$$C = \frac{n(n^3 + 1)}{2}$$

### Example

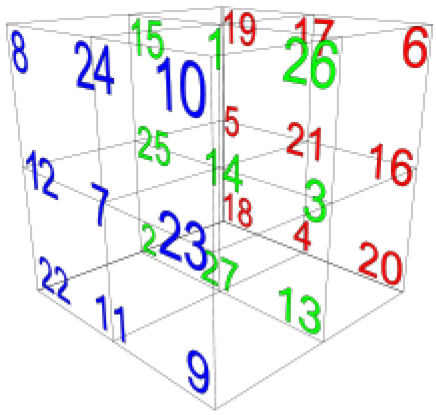



Figure 23 – Example of a 3x3x3 magic cube

Its magic constant is $\frac{n(n^3+1)}{2} = 126$, as can be verified, for example, with the following numbers:

— $8 + 24 + 10 + 12 + 7 + 23 + 22 + 11 + 9 = 126$
— $10 + 26 + 6 + 23 + 3 + 16 + 9 + 13 + 20 = 126$

# 7 Conclusion

Magic squares are intriguing numerical combinations in square grids with some interesting properties and probably even more mysterious properties yet to be discovered. They have been used for centuries all around the world. The question to be asked now is: 'What other secrets lie under this quadratic arrangement of numbers and how will this be useful in the future of mathematics?' Answers to this sort of question will remain an enigma until research sheds more light upon these indeed *magic squares*.